Intro to Lab: Modeling a Microvascular Network on a Chip

ECE 1810
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Lab Objectives

- To characterize simple model of microvascular system using microfluidic device
- To introduce you to microfluidics
  - Measure flow velocities within model network before and after a blockage (occlusion or clot of a blood vessel - stroke)
    - particle velocimetry
  - Show how EE concepts transfer to the design and analysis of a microfluidic network
  - Lab will be accompanied by an assignment
    - theoretically calculate pressure and velocity in each channel before and after blockage
    - compare calculated pressure and velocities with empirical velocities

What is Microfluidics: What's in a name?

- Infer: manipulation of minute quantities of fluids (L and G) … inside very small devices
- Applies ultra-precise fab technology to conventionally "messy" fields like biology (dishes, plates) and chemistry (vats, reaction vessels)
  - Emerged early 90s: Andreas Manz, George Whitesides, but idea has been around much longer:
    - Richard Feynman – "There's Plenty of Room at the Bottom" – 1959
  - Why?
    - Unique physics at microscale permits novel creations
    - Permits more orderly, systematic approach to bio-related problems, reduces physical effort:
      - drug discovery
      - cytotoxicity assays
      - protein crystallization

Background

- Aims to do for chemistry and biology what Kilby's and Noyce's invention of the IC did for the field of electronics:
  - dramatic reduction in size and cost
  - increase in performance and portability
  - lab on a chip: goal to have all stages of chemical analysis performed in integrated fashion:
    - sample prep.
    - rxns
    - analyte sep.
    - analyte purification
    - detection
    - analysis
  - µTAS, POCT, decentralized
- Benefits: integration of functionality, small consumption of expensive reagents, automation, high throughput, reproducibility of results, parallelization, disposable

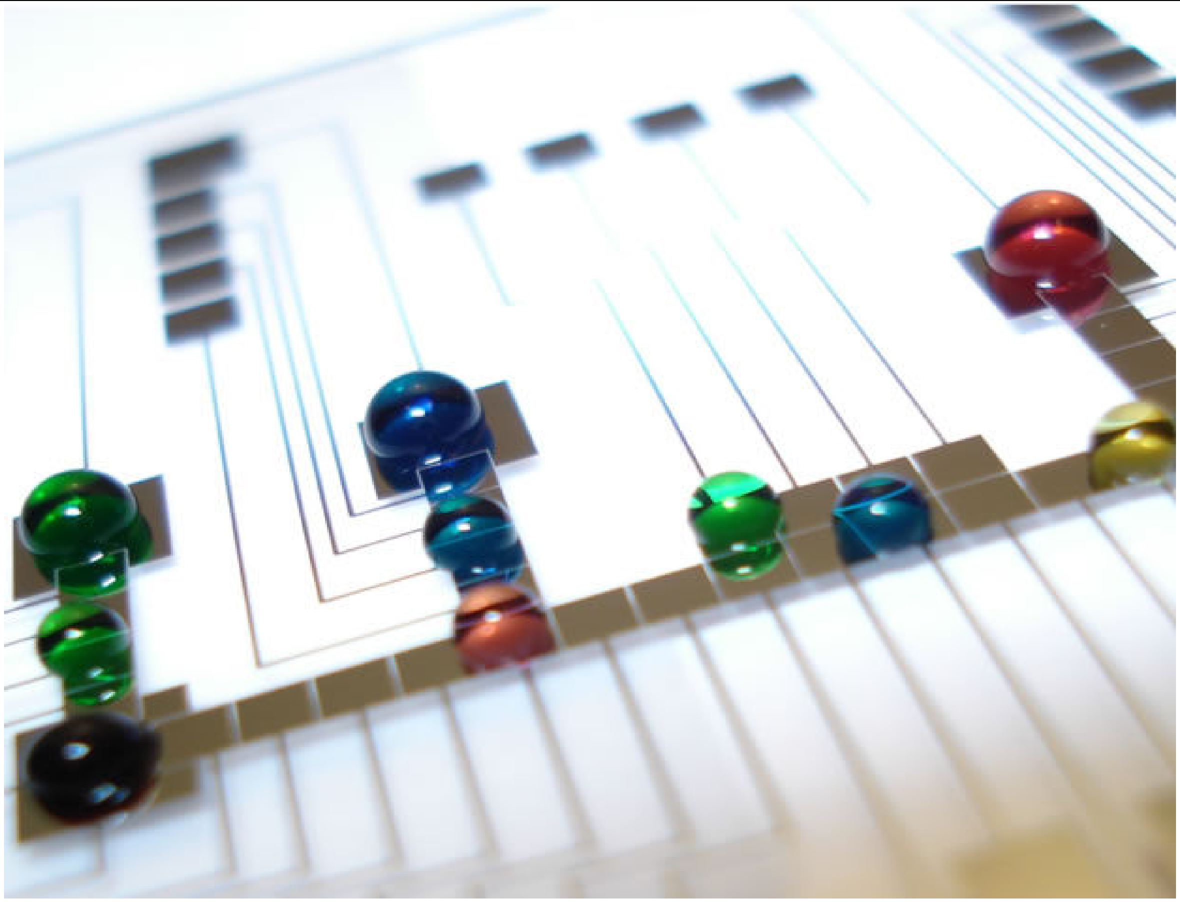
Defining Characteristics – Surface Area

- Compare surface area to volume ratio of cubes 1 cm^3, 100 μm^3, and 10 μm^3, 100nm^3, and 10nm^3 on a side:

side length (l)	side area (l^2)	surface area ($6l^2$)	volume (l^3)	SA/V ratio
0.01	0.0001	0.0006	0.000001	600
0.0001	0.00000001	0.00000006	1E-12	60000
0.00001	1E-10	6E-10	1E-15	600000
0.0000001	1E-14	6E-14	1E-21	60000000
0.00000001	1E-16	6E-16	1E-24	600000000

Defining Characteristics – Surface Area

- Surface effects become dominant:
  - Surface tension
    - Disparity in intermolecular forces among molecules in bulk and at interface – leads to formation of droplets or bubbles
  - Capillary action
    - Wicking of fluid through a capillary tube (microchannel) when adhesive attraction for walls > than cohesive forces among liquid molecules



Droplets on hydrophobic surface:
cohesive forces > adhesive forces
fluid beads into a droplet

Note: More pronounced in smaller tubes

Defining Characteristics – Laminar Flow

- Compare surface area to volume ratio of cubes 1 cm^3, 100 μm^3, and 10 μm^3, 100nm^3, and 10nm^3 on a side. Calculate the gravitational force acting on each cube assuming it is made of silicon with a density of 2.33 g/cm3:

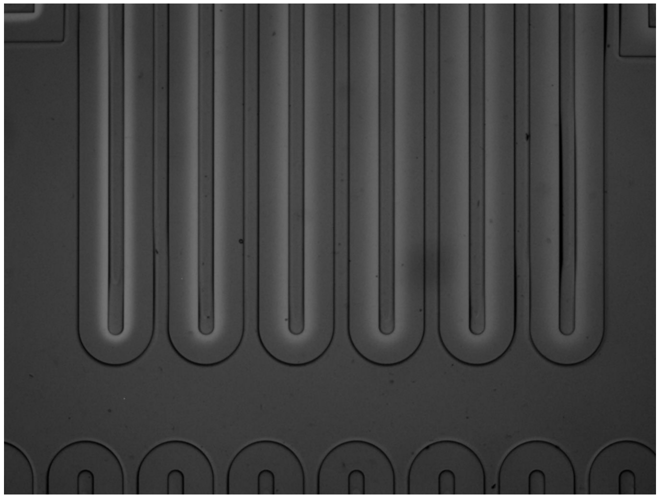
side length (l)	side area (l^2)	surface area ($6l^2$)	volume (l^3)	SA/V ratio	volume in cm^3	mass (g)	mass (kg)	F = ma (N)
0.01	0.0001	0.0006	0.000001	600	1	2.33	0.00233	0.022834
0.0001	0.00000001	0.00000006	1E-12	60000	0.000001	0.00000233	2.33E-09	2.2834E-08
0.00001	1E-10	6E-10	1E-15	600000	0.000000001	2.33E-09	2.33E-12	2.2834E-11
0.0000001	1E-14	6E-14	1E-21	60000000	1E-15	2.33E-15	2.33E-18	2.2834E-17
0.00000001	1E-16	6E-16	1E-24	600000000	1E-18	2.33E-18	2.33E-21	2.2834E-20

Conclusion: gravitational force is negligible at microscale when compared to other forces (like surface forces)

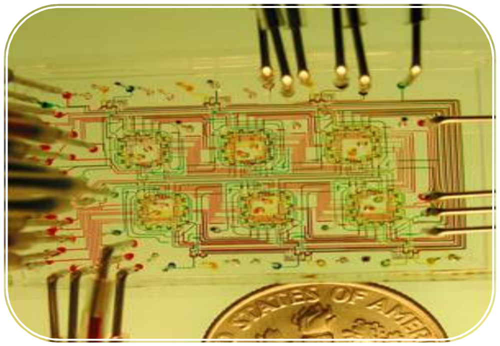
Defining Characteristics – Laminar Flow

Reynolds number – compares relative strengths of different forces (inertial vs. viscous)

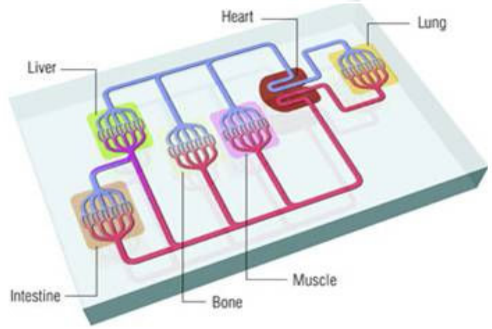
- small Re: laminar flow (<2300)
- large Re: turbulent flow (>2300)
- Re in most μfluidic devices << 1
  - Laminar flow is both an asset and hindrance:
    - Transport of objects is predictable
    - Mixing is difficult – two parallel streams will mix only across the interface via Fickian diffusion



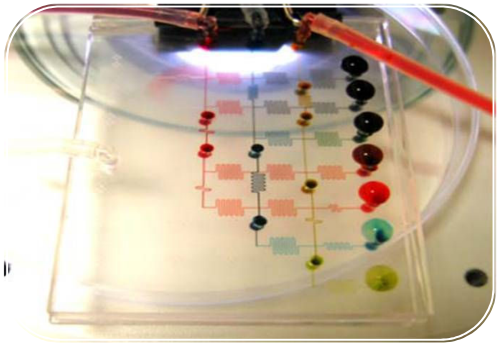
Examples of Microfluidics



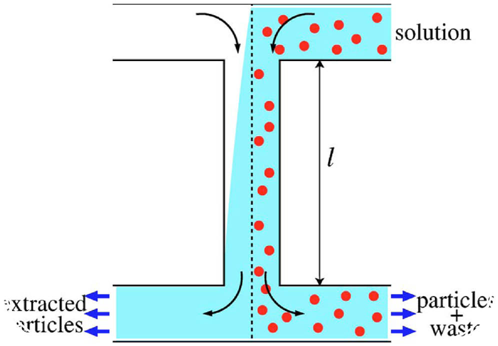
LSI



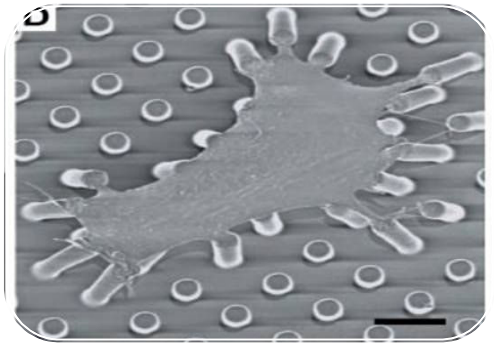
Body on chip



Concentration gradient



Filtering Without Membranes

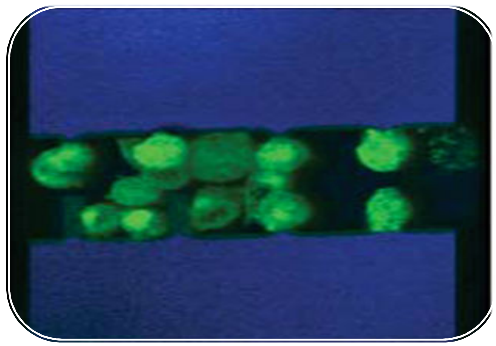


Traction force microscopy

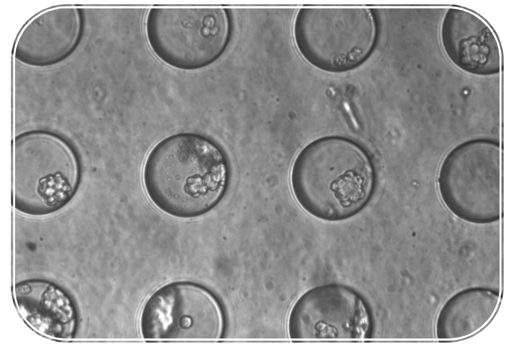
Portable medical diagnostics

Examples of Microfluidics

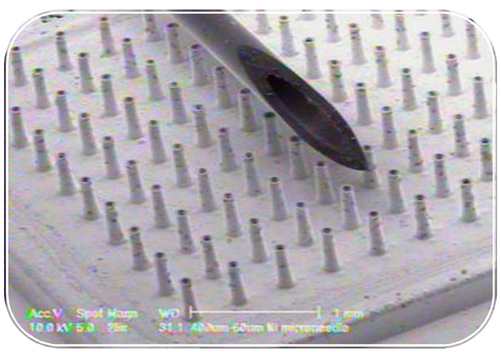
Lab on a CD / Centrifugal Microfluidics



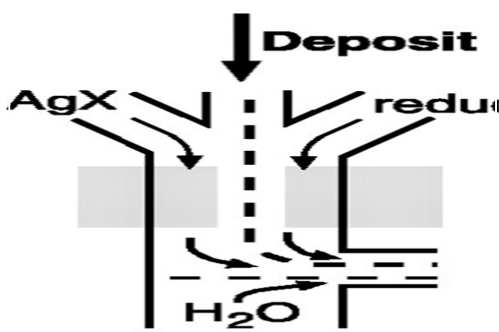
Cantilever-based biosensors



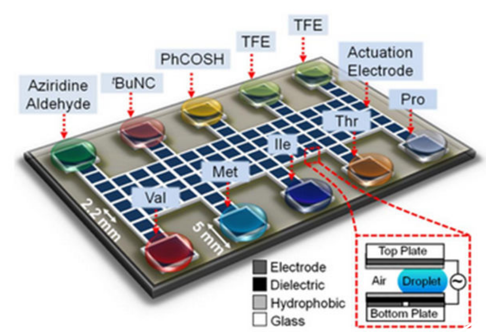
Biomimesis



Microneedle Array for Painless Drug Delivery



Microfabrication



Droplet Microfluidics

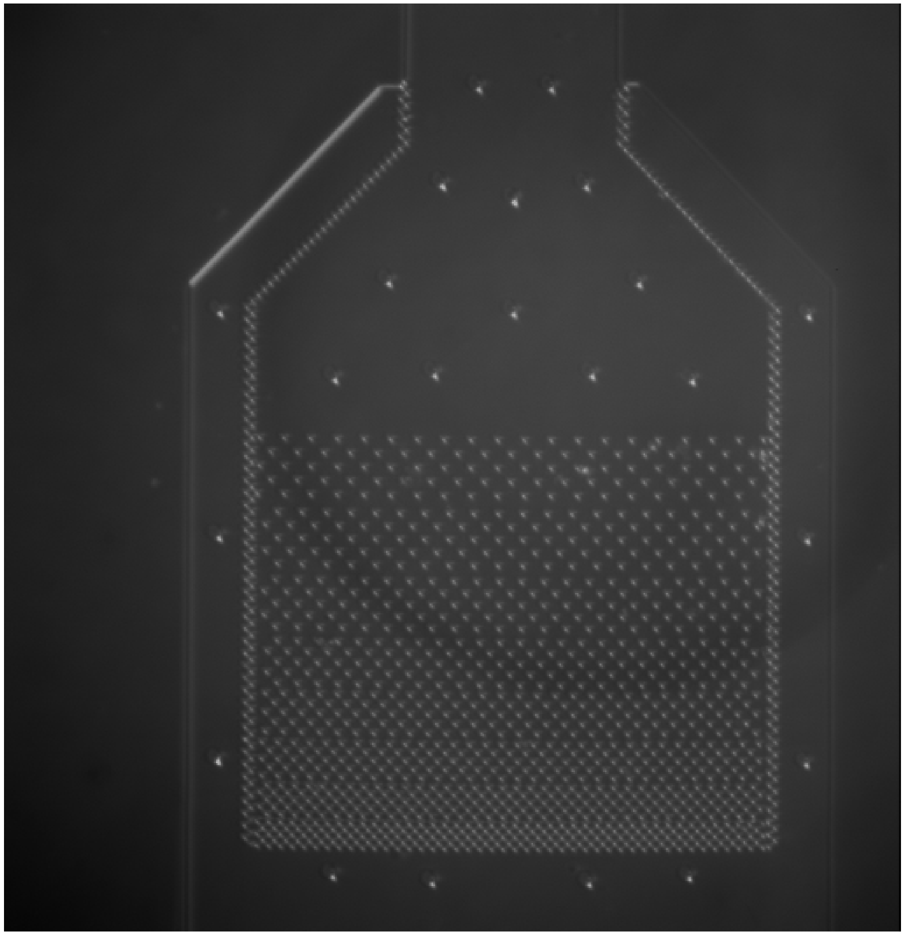
Field Overview

- Very low tech to very complex
- Single layer lithography to multiple layers requiring alignment
- Minimalistic (paper-based) vs. sophisticated
- Components vs. Systems (hierarchical)
- Devices vs. Applications

Components → Systems

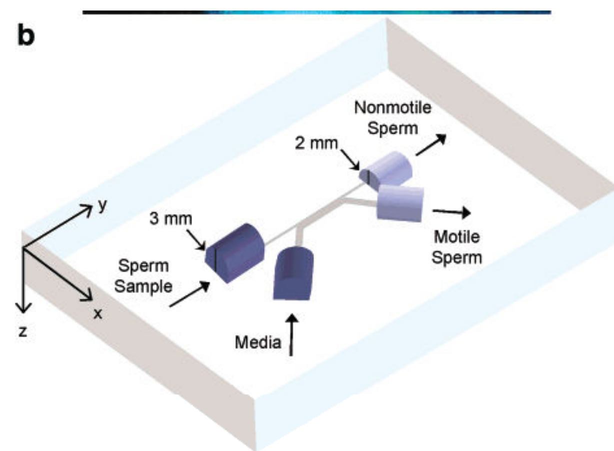
- Wide array of components: components together make up systems:
  - μreactors
  - μmixers
  - μheaters – (PCR, cell culture)
  - μfilters – cell capture
  - μvalves
  - μpumps

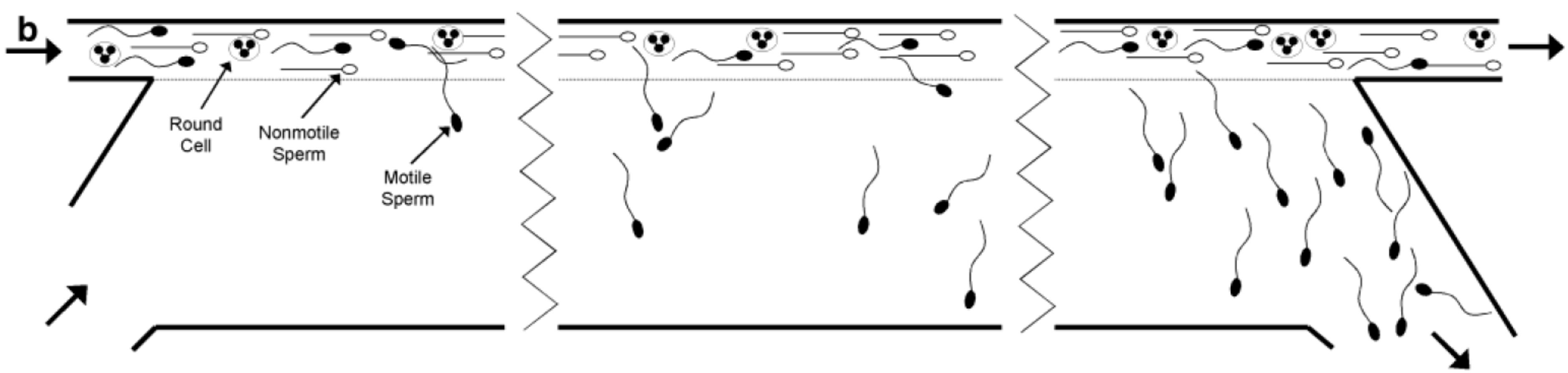
  Everything at macroscale has counterpart at microscale, but microscale physics permits novel components



Application 1: motile sperm sorter

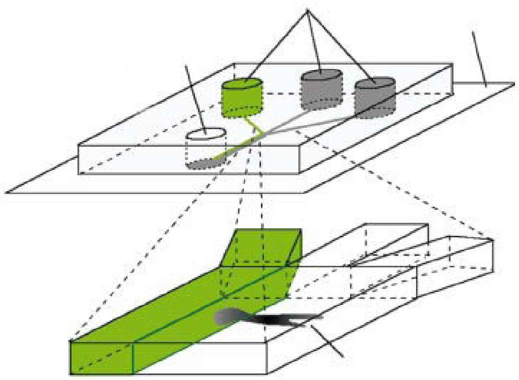
- Membraneless H-filter
  - low-Re number so mixing occurs by diffusion
  - Two streams of fluid will flow alongside each other
  - Exploit this to separate particles or cells based on differences in diffusivity or motility

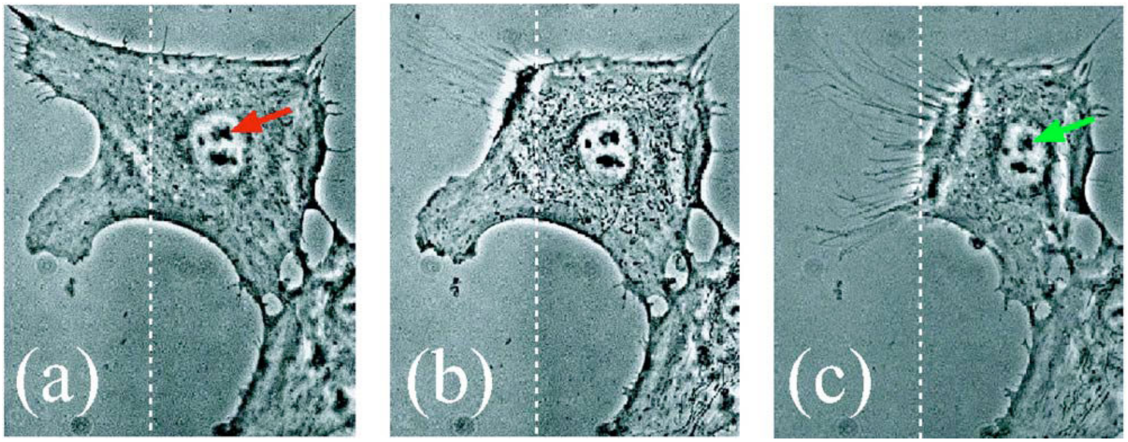




Application 2: laminar flow to study individual cell movement

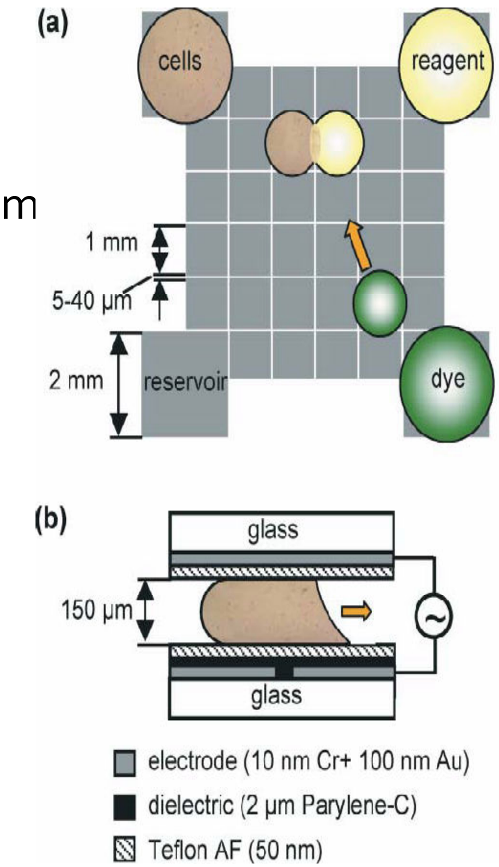
- Cellular response to stimuli can be probed
- Trypsin disrupts cell-substrate adhesion
  - causes cell to lose its hold and recoil to the trypsin-free stream

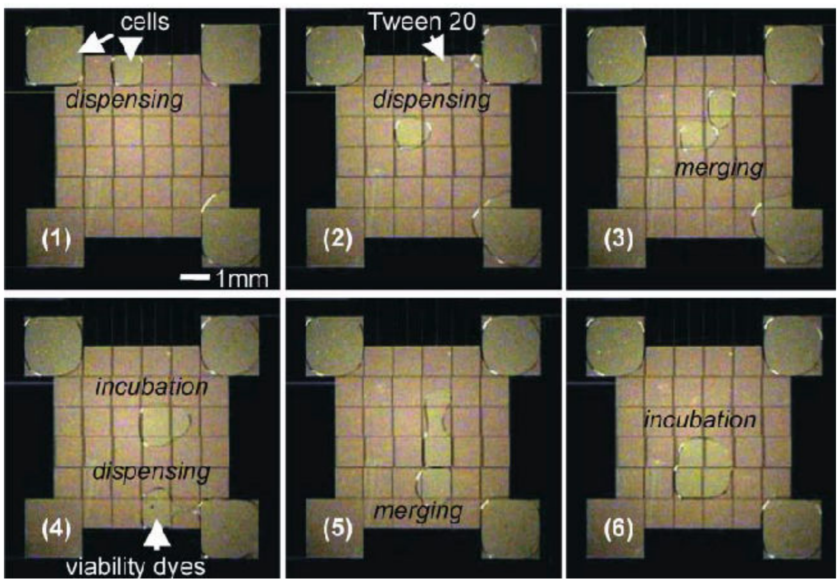




Application 3 – drug screening

- Each droplet can serve as a reaction vessel to study a cell's response to antibiotic
  - Encapsulate bacterium into droplet
  - Droplet containing viability indicator is merged with bacterium containing droplet
  - Droplet containing antibiotic is merged with this droplet, incubated, and intensity of fluorescent agent measured





Commercialization

- Despite being subject of ongoing research, microfluidics is being commercialized (startups, spinoffs, lucrative extensions to established)
- Microfabrication-related industries (MEMS, inkjet heads, accelerometers, gyroscopes) much more mature and profitable
  - No standards (DIP), competing paradigms, diff. materials
- Research has spawned microfluidics companies
  - high-throughput low-cost DNA sequencing
  - portable diagnostic tools
  - novel displays for E-readers

Commercialization

Table 1 Companies assigned microfluidic and microarray patents in the USA

Company	Assigned microfluidic and microarray patents (US only)
Caliper Lifesciences	215
Agilent	97
Affymetrix	73
Nanostream	34
Nanogen	20
GE Healthcare (formerly Amersham)	19
Zyomyx	16
Illumina	15
Gyros	14
Tecan	13
Honeywell	11
Advion	10
Cepheid	9
Beckman Coulter	8
Biacore	8
Micronics	8
Fluidigm	7
Qiagen	6
Sequenom	6
Aurora Biosciences	4
Micralyne	4
Bioforce Nanosciences	3
Eksigent Technologies, LLC	3
Evotec Technologies	3
Genomic Solutions	3
Genetix	3
Molecular Devices	3
Randox Laboratories	3
Innovadyne	2
Telechem International	2
Amnis Corporation	1
BioDot	1
Bio-Rad	1
BioTrove	1
Cellectricon	1
Combimatrix	1
Perkin Elmer	1
Shimadzu	1

Connection to EE

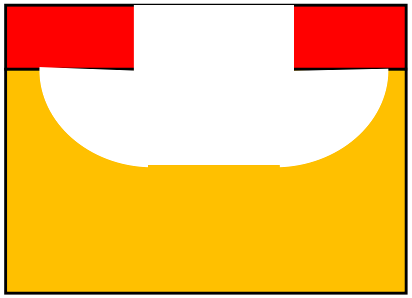
- Basic concepts have direct analogs to EE (voltage, current, resistance)
- Design and analysis of devices uses same methods used to analyze circuits (KCL, KVL, Ohm's Law)
- Fabrication of devices uses tools and techniques borrowed from semiconductor industry
- Entire electronic devices have counterparts in microfluidics:
  - shift register
  - voltage divider (drop pressure)
  - current divider (split flow)
- Control is effected in same way:
  - multiplexing
  - truth tables

Fabrication: How Are Microfluidic Devices Made?

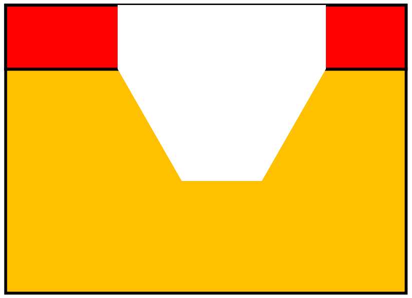
- Semiconductor fabrication methods
- Pattern is defined on surface (2D) and then transferred into vertical plane
- Process flow – depicted as a cross-sectional view, each step showing execution of one step (addition of layer, exposure, removal of layer, etc.)
- Detailed process to create master, replicas created inexpensively

Fabrication: How Are Microfluidic Devices Made?

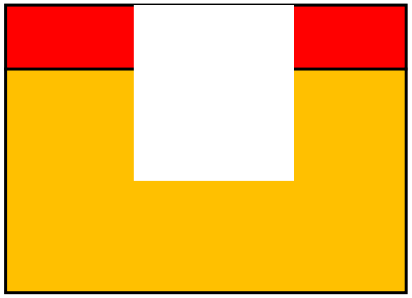
- Top-down vs. bottom-up
- Bottom-up:
  - μcontact printing for SAM
- Top-down:
  - Bulk (subtractive)
    - Isotropic etching
      - wet
    - Anisotropic etching
      - dry
    - Laser ablation
    - CMP
  - Surface (additive – structural and sacrificial layers)
    - spin coating
    - electroplating
    - oxidation
    - chemical vapor dep. – evaporation
    - physical vapor dep. – sputtering



Isotropic (HF)



Directional Crystal planes (KOH)



Anisotropic (RIE)

Process Flow: Soft-Lithography

1) Spin SU-8

2) Expose

3) Develop

4) Apply Anti-stick monolayer

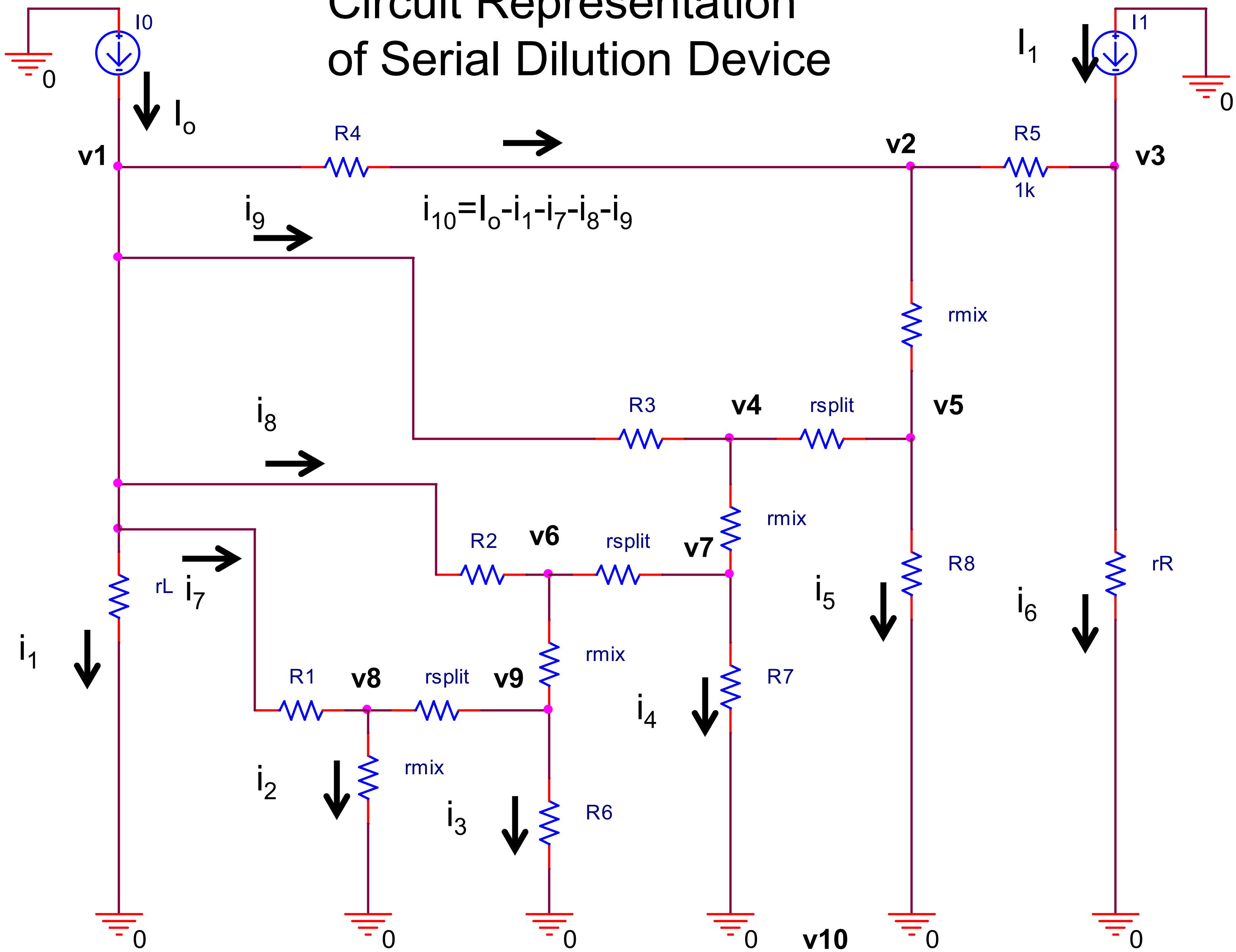
5) Cast PDMS

6) Plasma treat + bond

Example: Serial Dilution Gradient Generator

- Works on laboratory practice of serial dilution:
  - Create a number of solutions, each with a unique concentration, by using a small amount of the more concentrated solution to create the next less concentrated solution:
  - More reliable
  - Log base 10 Example:
    - Stock solution consists of 1mL of solute in 9mL of solvent = 1X
    - 1mL of this into 9mL solvent yields 0.1X
    - 1mL of this into 9mL solvent yields 0.01X
- In a microfluidic implementation, the concentration of each step is determined by the ratio of two merging flow rates at the junction of the buffer and the agent
- After the chemical to be diluted is merged with the buffer at the intersection, the combined solution is passed through the mixing region
- Some portion of the diluted solution is collected at the outlet and remainder is passed on to the next intersection

Circuit Representation of Serial Dilution Device



Serial Dilution: Log Base 10 Profile

System of Equations to Solve (42 equations in 33 variables):

1. $v_1/rL+(v_1-v_2)/r_4+(v_1-v_4)/r_3+(v_1-v_6)/r_2+(v_1-v_8)/r_1-I_o = 0$
2. $(v_2-v_1)/r_4+(v_2-v_5)/r_{mix}+(v_2-v_3)/r_5 = 0$
3. $-I_1+(v_3-v_2)/r_5+v_3/rR = 0$
4. $(v_4-v_1)/r_3+(v_4-v_5)/r_{split}+(v_4-v_7)/r_{mix} = 0$
5. $(v_5-v_4)/r_{split}+(v_5-v_2)/r_{mix}+v_5/r_8 = 0$
6. $(v_6-v_1)/r_2+(v_6-v_7)/r_{split}+(v_6-v_9)/r_{mix} = 0$
7. $(v_7-v_6)/r_{split}+(v_7-v_4)/r_{mix}+v_7/r_7 = 0$
8. $(v_8-v_1)/r_1+(v_8-v_9)/r_{split}+v_8/r_{mix} = 0$
9. $(v_9-v_8)/r_{split}+(v_9-v_6)/r_{mix}+v_9/r_6 = 0$
10. $I_o+I_1 = i_1+i_2+i_3+i_4+i_5+i_6$
11. $i_{10} = I_o-i_1-i_7-i_8-i_9$
12. $v_1-v_2 = i_{10}*r_4$
13. $v_1-v_4 = i_9*r_3$
14. $v_1-v_6 = i_8*r_2$
15. $v_1-v_8 = i_7*r_1$
16. $v_1 = i_1*rL$
17. $v_3-v_2 = (I_1-i_6)*r_5$
18. $v_2-v_5 = (i_{10}+I_1-i_6)*r_{mix}$
19. $v_5 = i_5*r_8$
20. $v_5-v_4 = (i_{10}+I_1-i_6-i_5)*r_{split}$
21. $v_4-v_7 = (i_{10}+I_1-i_6-i_5+i_9)*r_{mix}$
22. $v_7 = i_4*r_7$
23. $v_7-v_6 = (i_{10}+I_1-i_6-i_5+i_9-i_4)*r_{split}$
24. $v_6-v_9 = (i_{10}+I_1-i_6-i_5+i_9-i_4+i_8)*r_{mix}$
25. $v_9 = i_3*r_6$
26. $v_9-v_8 = (i_{10}+I_1-i_6-i_5+i_9-i_4+i_8-i_3)*r_{split}$
27. $v_8 = i_2*r_{mix}$
28. $i_2 = i_7+i_{10}+I_1-i_6-i_5+i_9-i_4+i_8-i_3$
29. $i_1 = i_2$
30. $i_1 = i_3$
31. $i_1 = i_4$
32. $i_1 = i_5$
33. $i_1 = i_6$
34. $0.0001 = ((i_2-i_7)/i_2)*0.001$
35. $0.001 = ((i_{10}+I_1-i_6-i_5+i_9-i_4)/(i_{10}+I_1-i_6-i_5+i_9-i_4+i_8))*0.01$
36. $0.01 = ((i_{10}+I_1-i_6-i_5)/(i_{10}+I_1-i_6-i_5+i_9))*0.1$
37. $0.1 = ((I_1-i_6)/(i_{10}+I_1-i_6))*1$
38. $r_4 = 10000$
39. $r_5 = 2500$
40. $r_{split} = 2500$
41. $r_{mix} = 31446.54$
42. $I_o = 1$

Node Voltage Equations
KCL
Ohm's Law
Equations stipulating flow rates of all six concentrations be equal
Equations specifying concentrations
Selected values

Serial Dilution: Log Base 10 Profile

MATLAB Results:

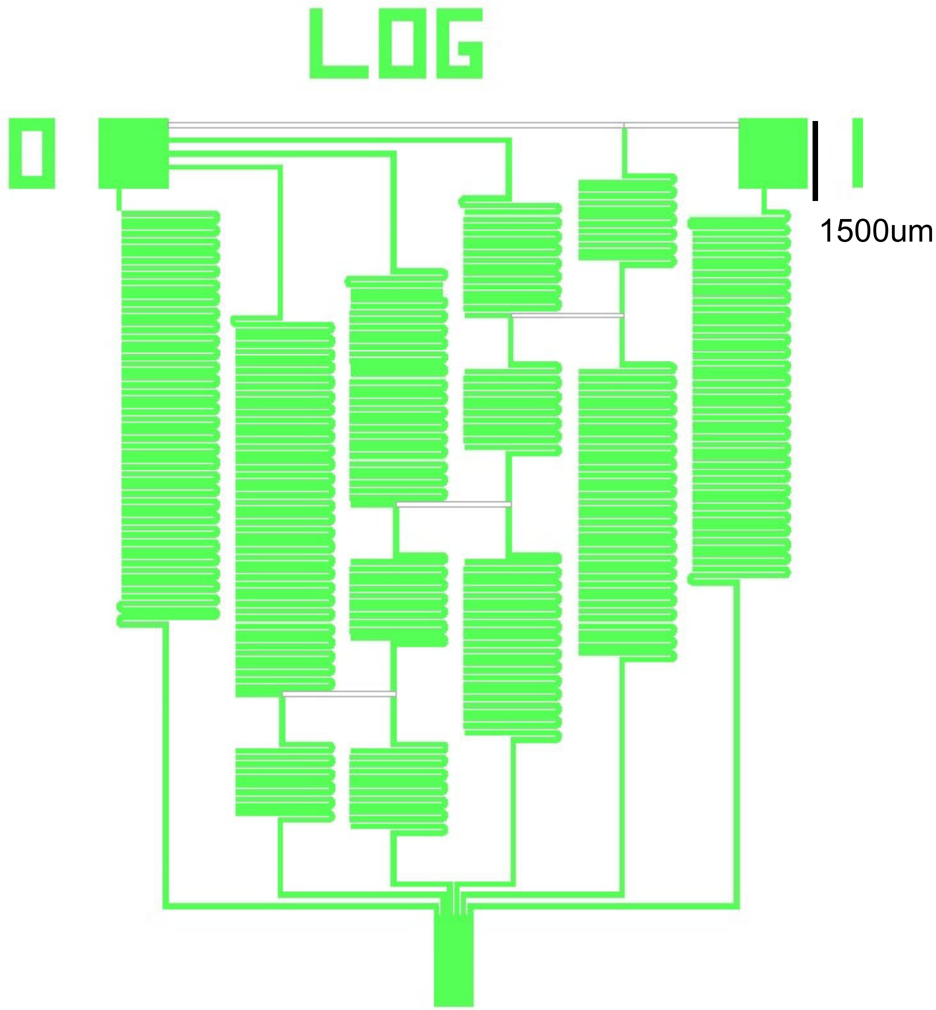
Having specified:

R_{mix} = 31446.54
R_{split} = 2500
R_4 = 10000
R_5 = 2500
I_o = 1
C's = 1,0.1,0.01,0.001,0.0001,0

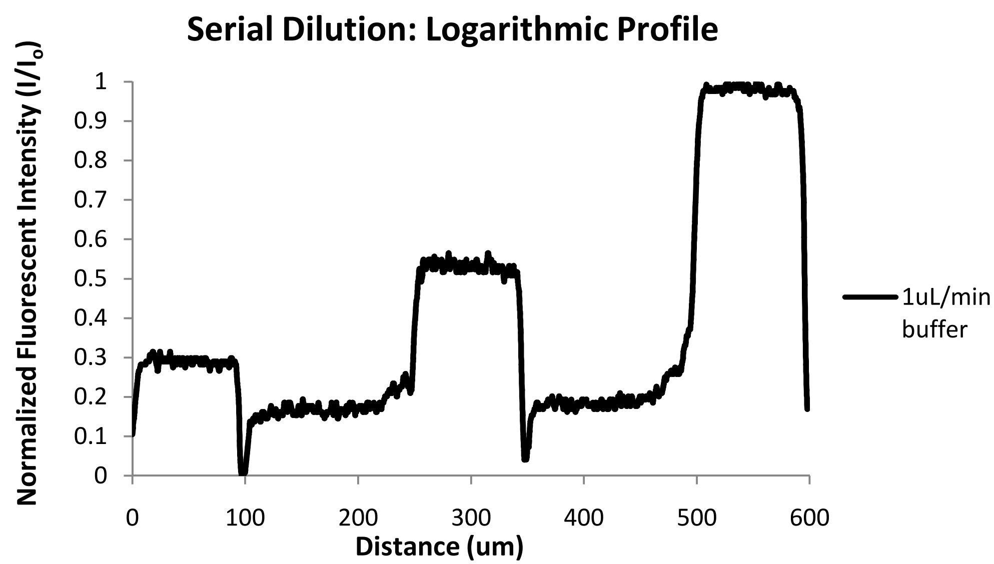
Yields:
I_1 = 0.2273

component	length (um)	multiple of r_{mix}
rL	146682.00	4.66
rR	136960.75	4.36
r1	128039.40	4.07
r2	81206.33	2.58
r3	45258.86	1.44
r4	10000.00	0.32
r5	2500.00	0.08
r6	31696.54	1.01
r7	66562.73	2.12
r8	101745.89	3.24
rmix	31446.54	1.00
rsplit	2500.00	0.08

Serial Dilution: Log Base 10 Profile CAD Image



Experimental Results: Log Base 10 Profile



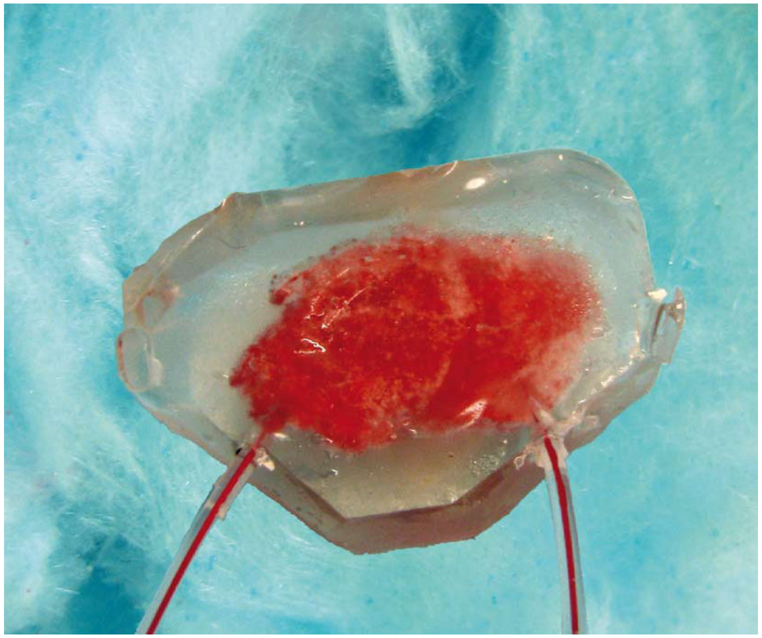
Grayscale fluorescent image of three most concentrated solutions, 40x magnification, ISO400, 10s, 35uM fluoroscein solution,1uL/min buffer flow rate

Biological Relevance

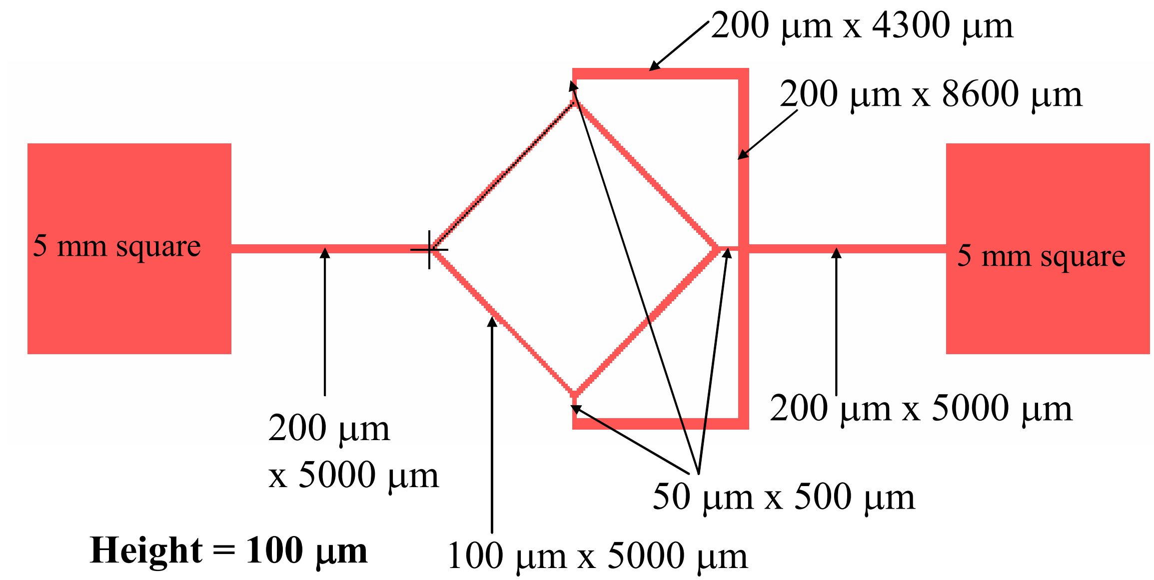
- Stroke
  - Interruption in blood supply to the brain caused by ischemia (insufficient blood flow) due to a blockage (blood clot, thrombus, embolism)
- Atherosclerosis
  - Narrowing of blood vessels due to build-up of fatty deposits
  - Leads to hypertension, which in turn exacerbates atherosclerosis
    - High blood pressure causes distension of vessels, which damages endothelium lining, which in turn attracts more fatty deposits

Microfluidic analogs of vascular system

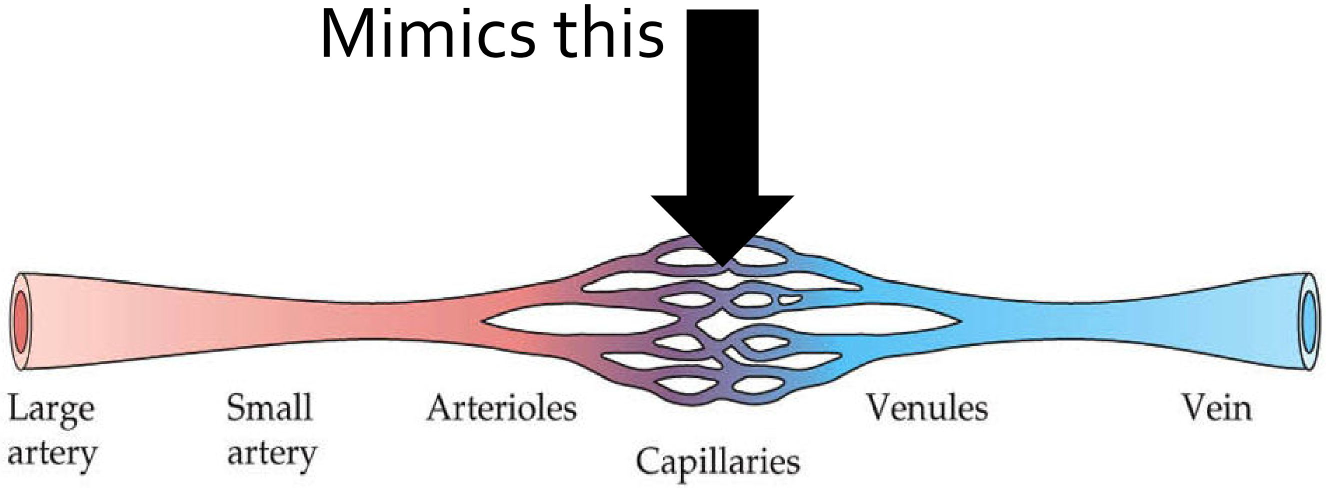
- 2008 – Harvard – investigated process of vasoocclusion (in sickle cell anemia patients) looking at geometrical, physical, chemical, biological factors that contribute
  - Channel width, total pressure difference across network, oxygen levels
  - Timed how long before an occlusion occurred
- 2008 – Cornell – created 3D vascular network using strands of molten sugar (cotton candy)
  - Sugar as sacrificial layer



Your Microfluidic Device



Mimics this

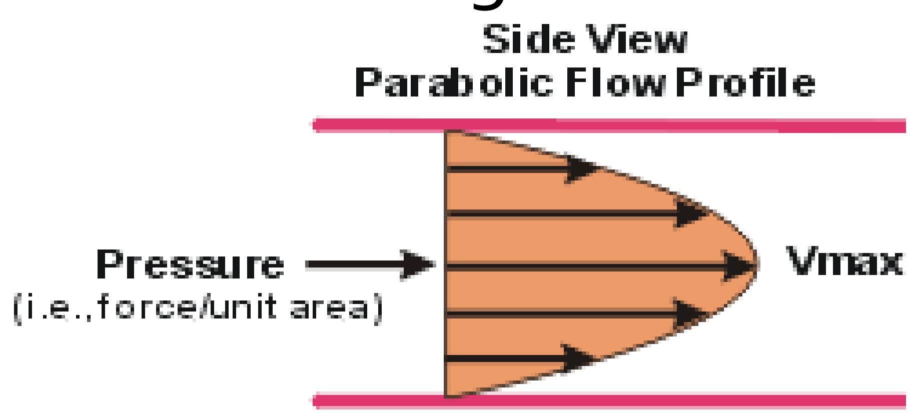


Particle Velocimetry

- Technique to qualitatively visualize and quantitatively assess fluid flow
  - Small tracer particles (beads) are added to fluid
  - Velocity of beads assumed to be same as velocity of surrounding fluid
  - Special buffer solution used to prevent beads from aggregating / adhering to channel surfaces

Note on Bead Motion

- In pressure-driven flow, velocity profile of fluid is parabolic
- v max in center
- v min near walls
- Be systematic: observe same location or take same number of beads from each location before and after blockage!



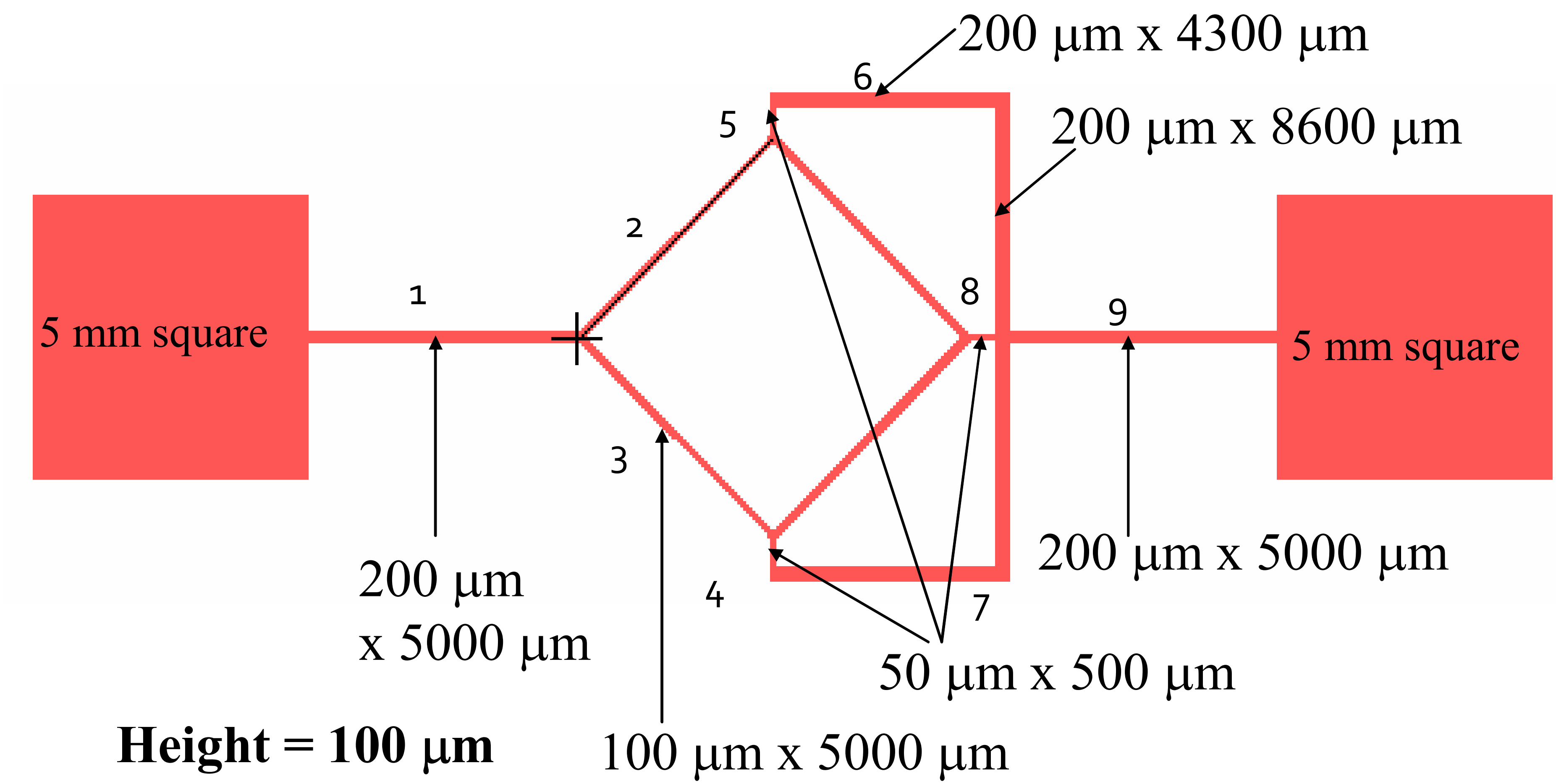
Procedure: Before Clot

- PDMS devices made for you
- Fill syringe with bead (10um dia.) solution
- Connect needle to syringe
- Connect tubing to needle
- Place syringe in syringe pump
- Flow @ 50uL/min to flush out bubbles
- Reduce flow rate to 2uL/min
- Capture series of images to track beads in each channel of device
  - V = d/t

Procedure: Clot

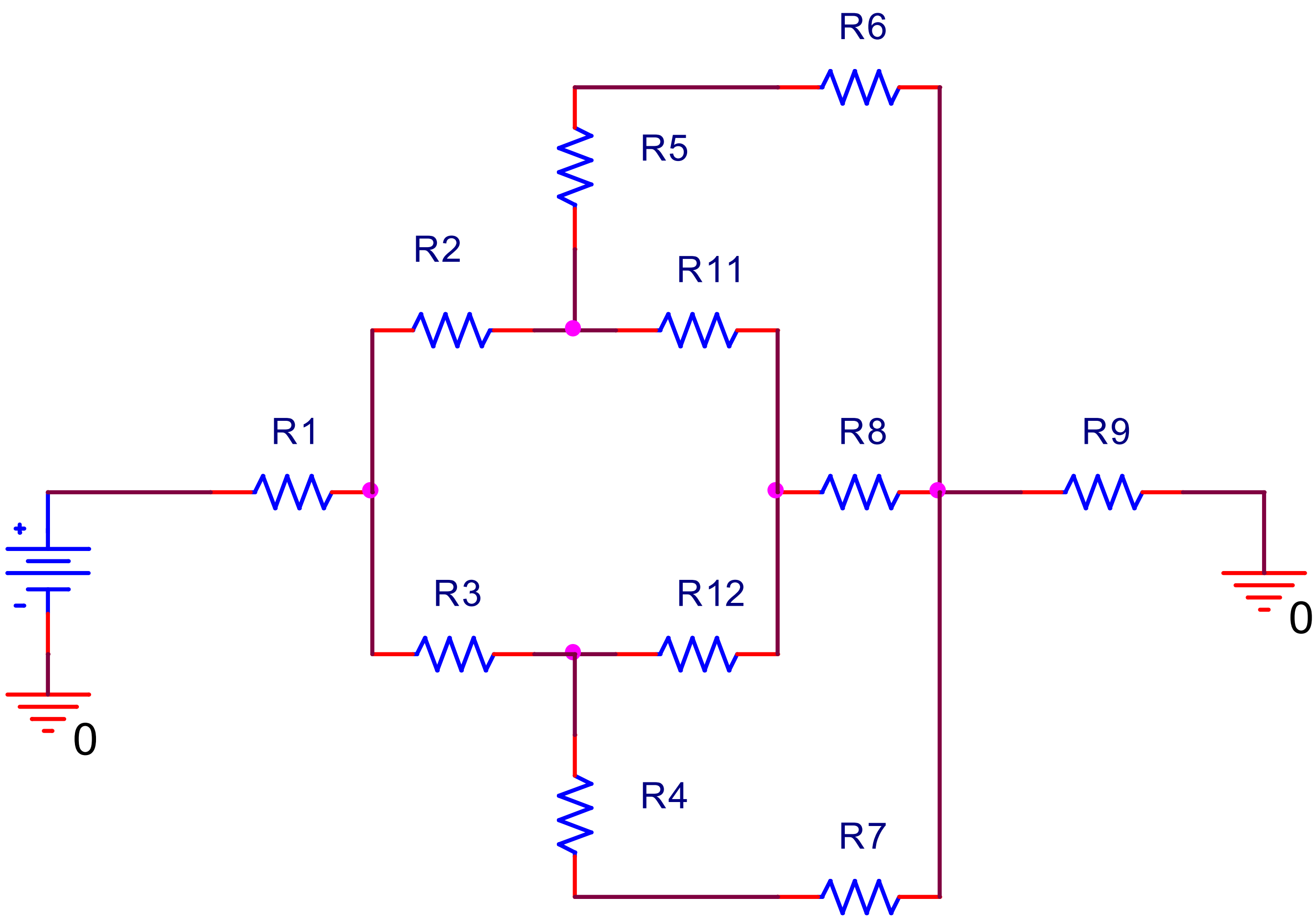
- Disconnect device from pump
- Using empty syringe, introduce air to dry device
- Punch hole in desired channel with punch
- Remove plug of PDMS
- Inject sealant to block channel
- Cure @ 60C for 10min
- Repeat process: track beads in each channel

How Many Channels?



Height = 100 µm

Circuit representation of microvascular chip



Results and Data Analysis Expected

- Average velocity for each channel before blockage
- Average velocity for each channel after blockage
- Prediction of flow pattern after channel blockage
- Bead Motion Analysis:
  - Want to track at least 10 beads in each channel
  - Track beads in each channel over several frames
  - Not interested in instantaneous velocity of beads (changes from frame to frame)
  - Interested in average velocity (total distance traveled / time observed)
  - Bin data to generate histogram (data partitioned into intervals and frequency of occurrence plotted)

Report Guidelines – SHEN/BRUCE

Background

- In analyzing a microfluidic device, it is useful to make an analogy to an electrical circuit:
  - Pressure → Voltage (Ground = reference potential → same pressure)
  - Flow Rate → Current
  - Hydraulic Resistance → Electrical Resistance (each channel gets represented by resistor)
    - $R=C_{geometry}*L/A^2$
    - For channel with square cross section:

      - R = 12uL/[(w^2)(h^2)]

      Special cases:

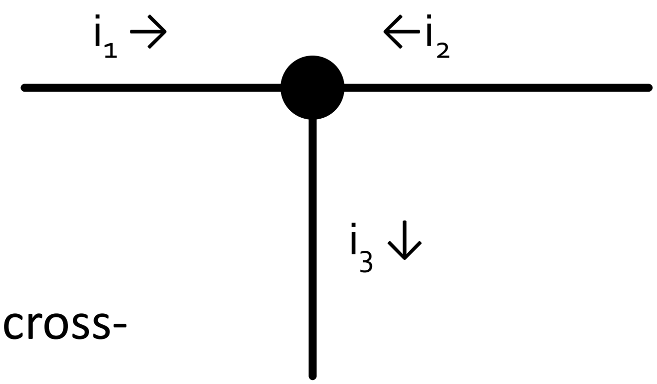
      Open circuit = infinite resistance

      Short circuit = zero resistance

      In this lab, blocking a channel is equivalent to an open circuit = zero flow

Analysis: Circuit Analog

- Can employ circuit analysis techniques and rules like Ohm’s law, KCL, KVL to facilitate analysis
- Ohm’s Law:
  - V = iR
  - Pressure = Volumetric Flow Rate * Channel Resistance
  - Volumetric flow rate = average linear velocity * cross-sectional area of channel
- KCL:
  - Conservation of charge → conservation of mass:
    - Flow rate into a node MUST equal flow rate out of that node
    - Nothing collects at node
- KVL:
  - Conservation of energy
  - Sum of voltage drops around a closed loop equals zero



Convention:
Define flow into node as + and flow out of node as –
$I_1 + i_2 – i_3 = 0$
$1_3 = i_1+i_2$

Pressure Drop Along Channel

Equation valid only if w >>h

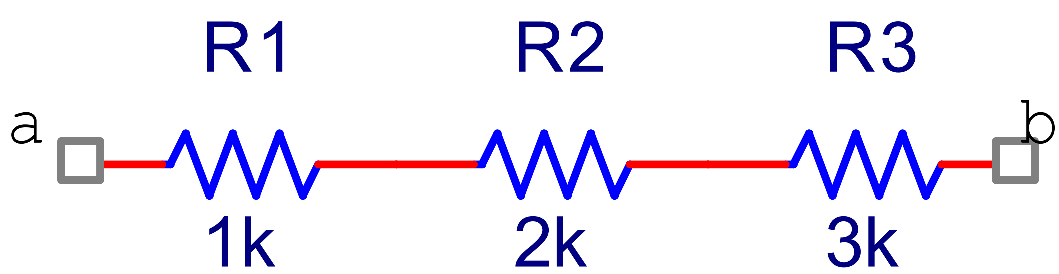
$$\Delta P = (\frac{-12\mu Q L}{wh^3}) \mapsto rectangular\ channel$$

ΔP – *pressure drop*
μ – *dynamic viscosity*
Q – *volume flow rate*
L – *channel length*
w – *channel width*
h – *channel height*

Circuit Analysis

- Resistances are either in series or parallel, except for when they're not

Resistors in Series



$R_T = R_1 + R_2 + R_3 =$
$1000+2000+3000 =$
6000 ohm

Sum is ALWAYS greater than any single resistance

Same current through each one

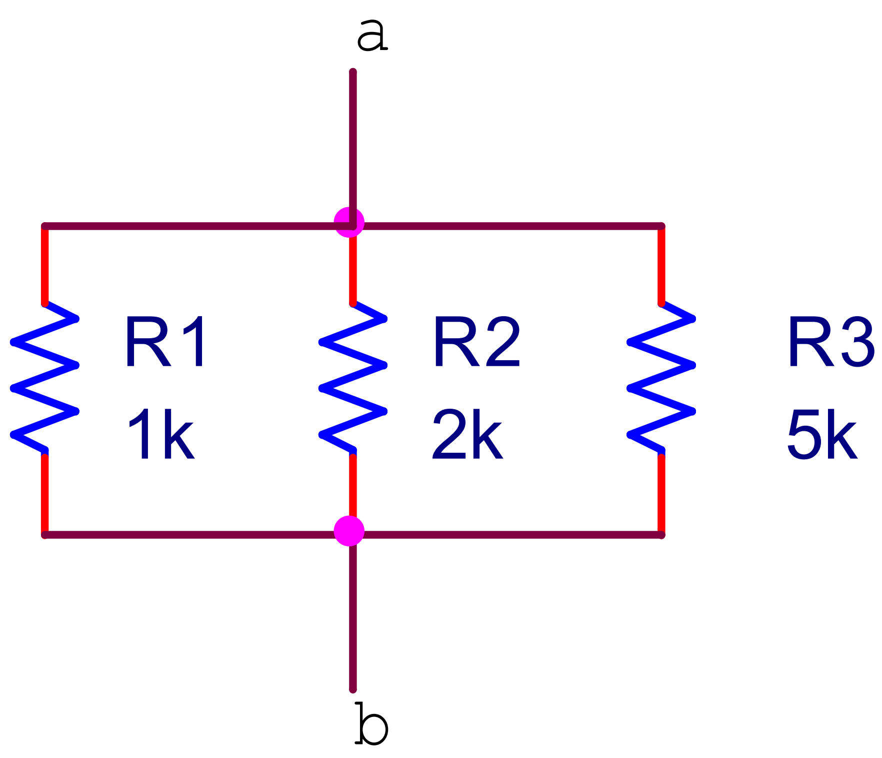
Resistors in Parallel

$R_T = 1 / (1/R_1 + 1/R_2 + 1/R_3) =$
$1/ (1/1000 + 1/2000 + 1/3000)$
$= 545$ ohm

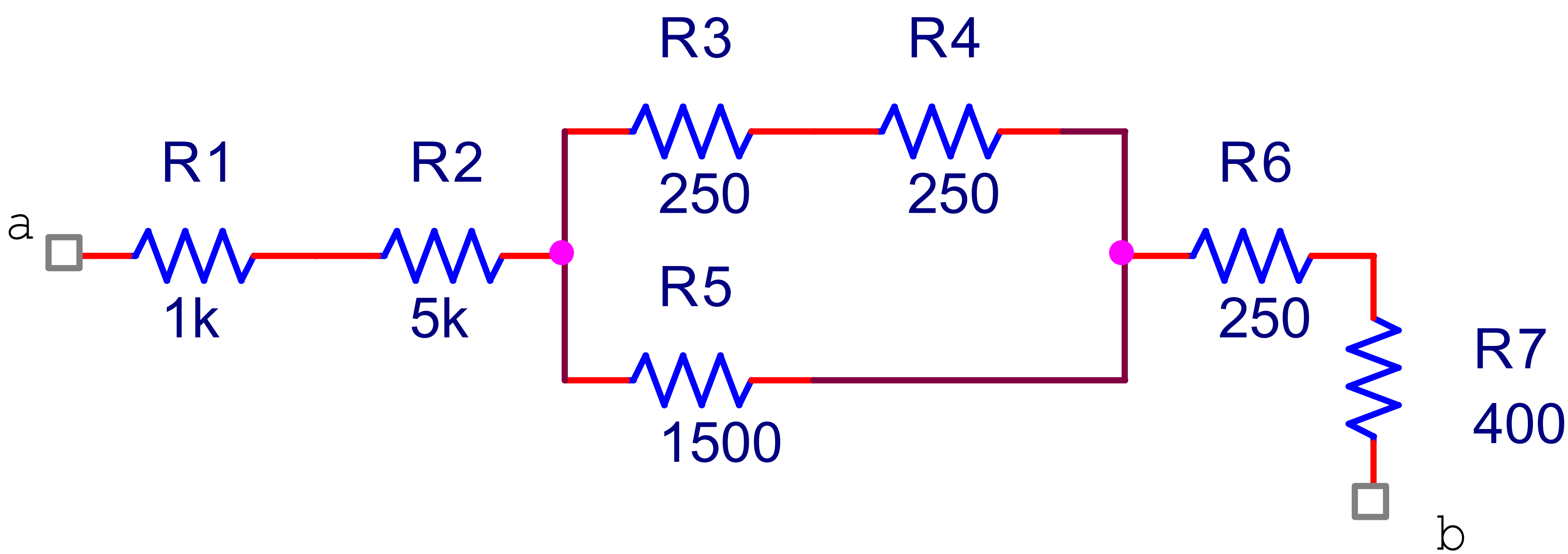
Sum is ALWAYS less than any single resistance

Special Case: 2 in parallel:
$R_T = R_1 * R_2 / (R_1 + R_2)$

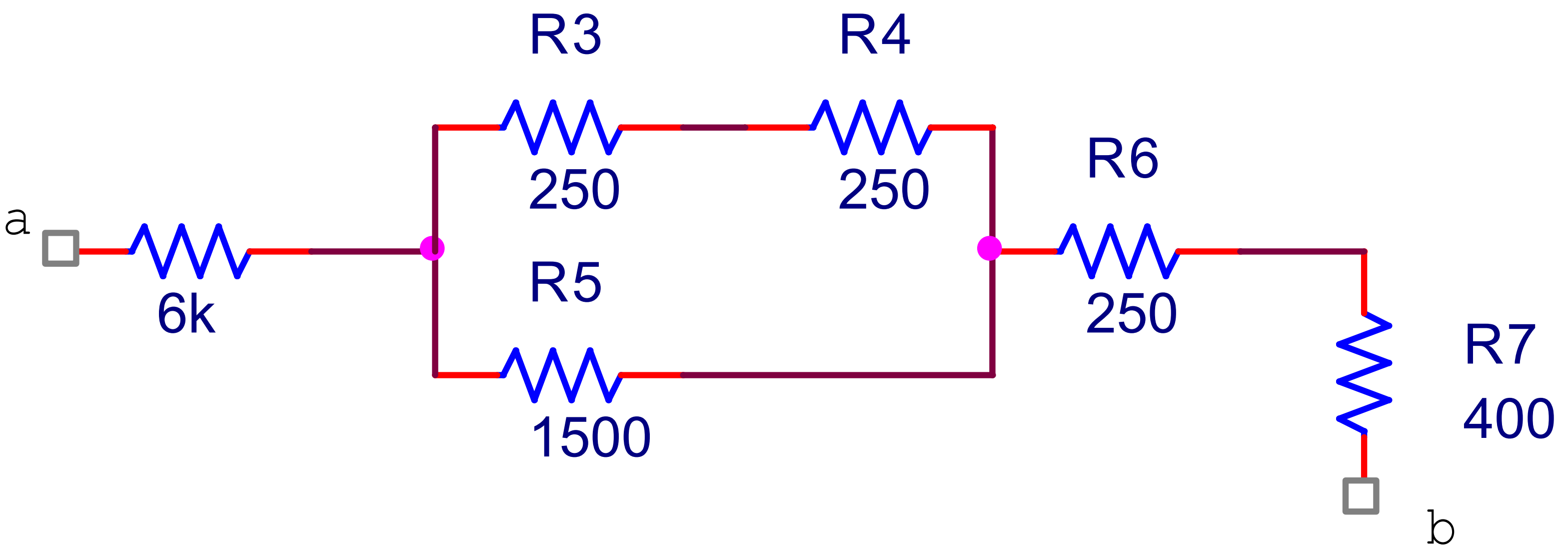
Same voltage across each one



Example: Resistance Network

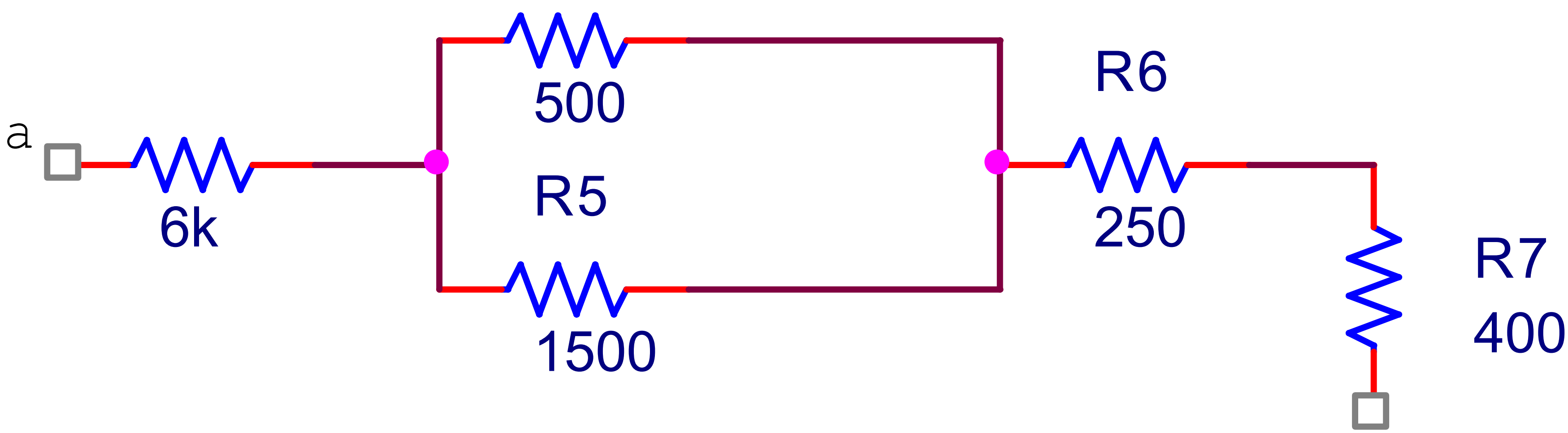


[redacted]



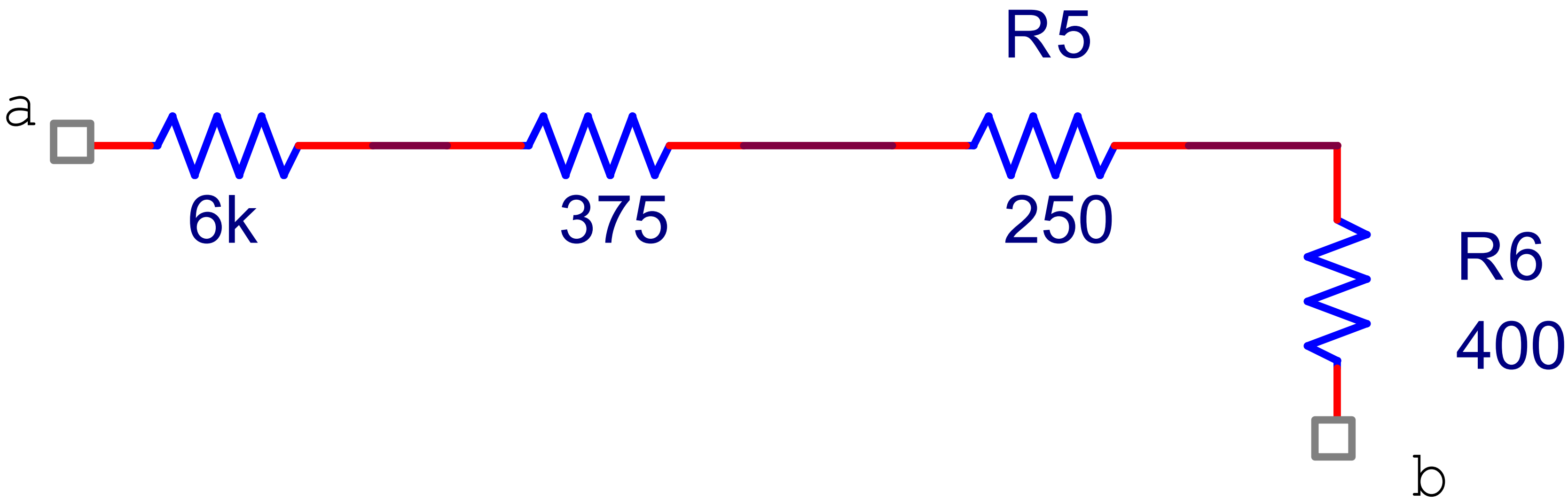
1. R1 and R2 in series = 6k

[redacted]



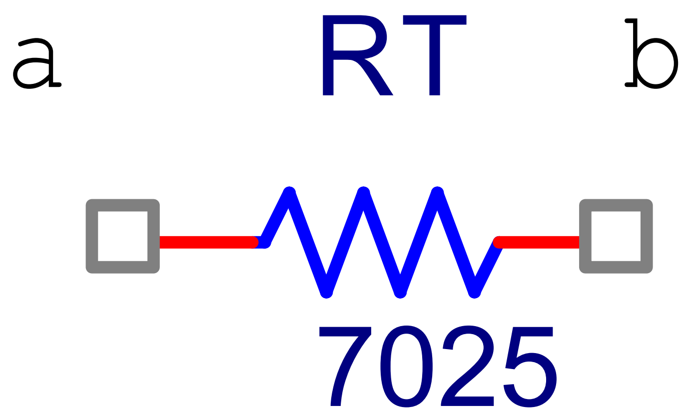
2. R_3 and R_4 in series = 500 ohm

[redacted]



3. 500 ohm in parallel with 1500 ohm = 375 ohm

[redacted]

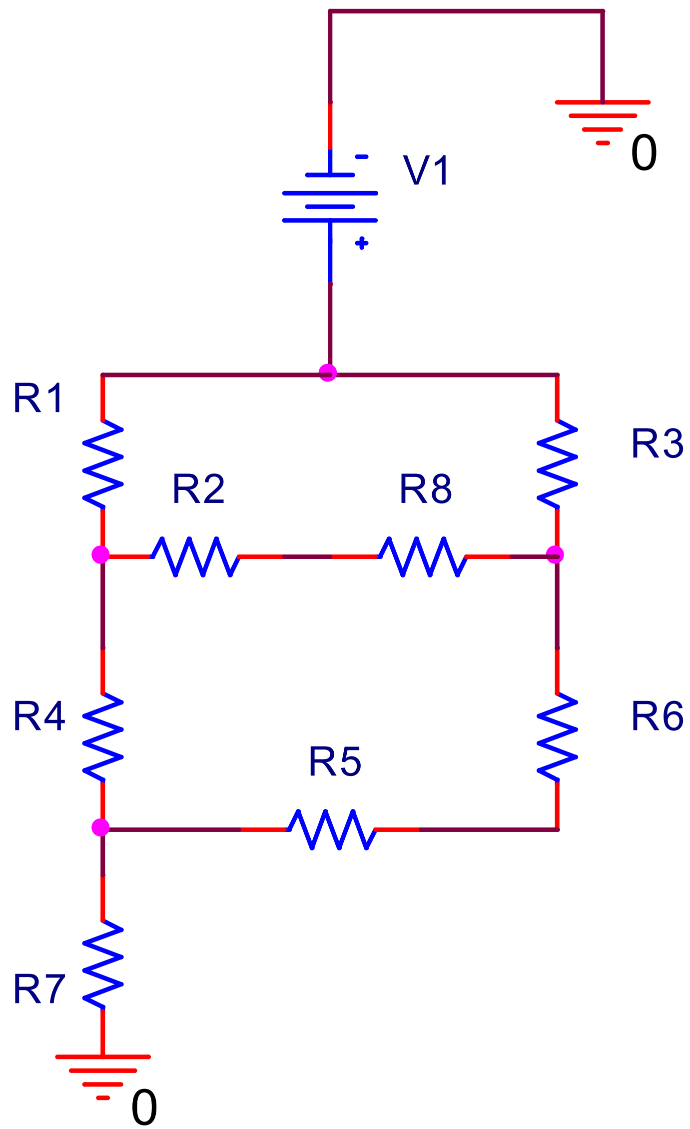


Remaining resistors in series = 6000 + 375 + 250 + 400 = 7025 ohm

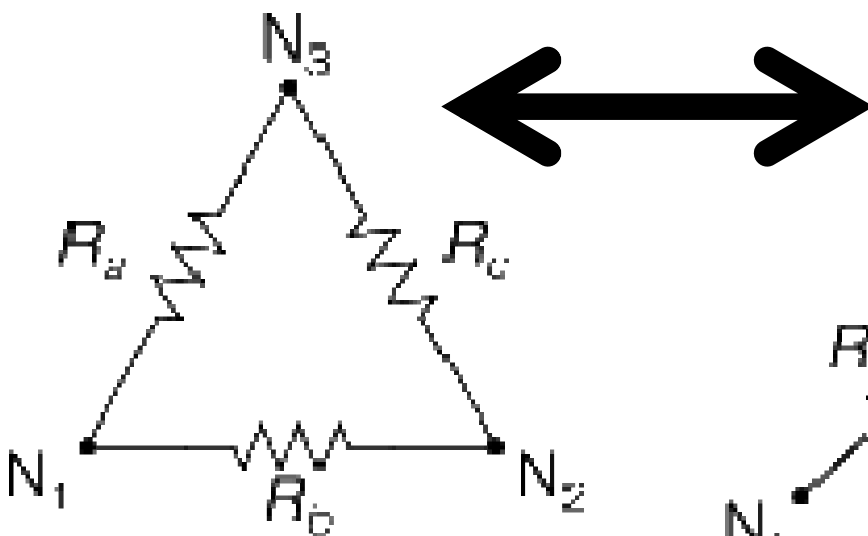
Special Case: Delta-to-Wye (Δ-toY) Transformation

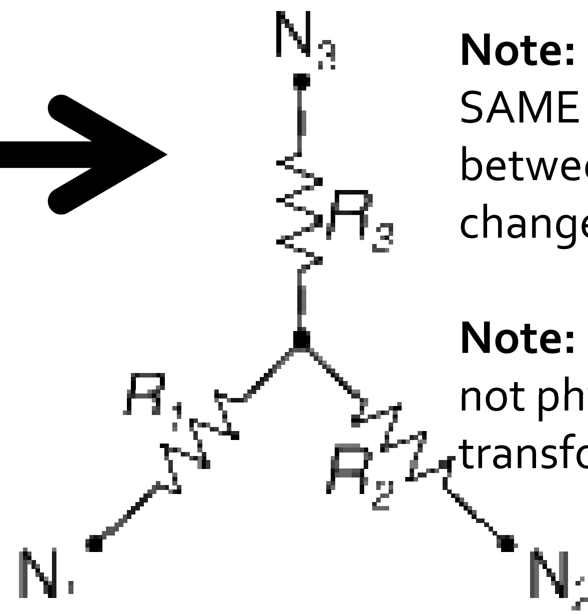
- Used when resistors are neither in series nor parallel
- How to identify this case:
  - Sometimes two resistors look like they are in parallel but they really aren't
  - Ask: do they share nodes at both terminals?
    - If yes, they are in parallel
    - If no, they aren't

Delta-to-Wye (Δ-toY) Transformation



Delta-to-Wye (Δ-toY) Transformation





Note: Nodes remain SAME , connections between nodes changes

Note: Mathematical not physical transformation

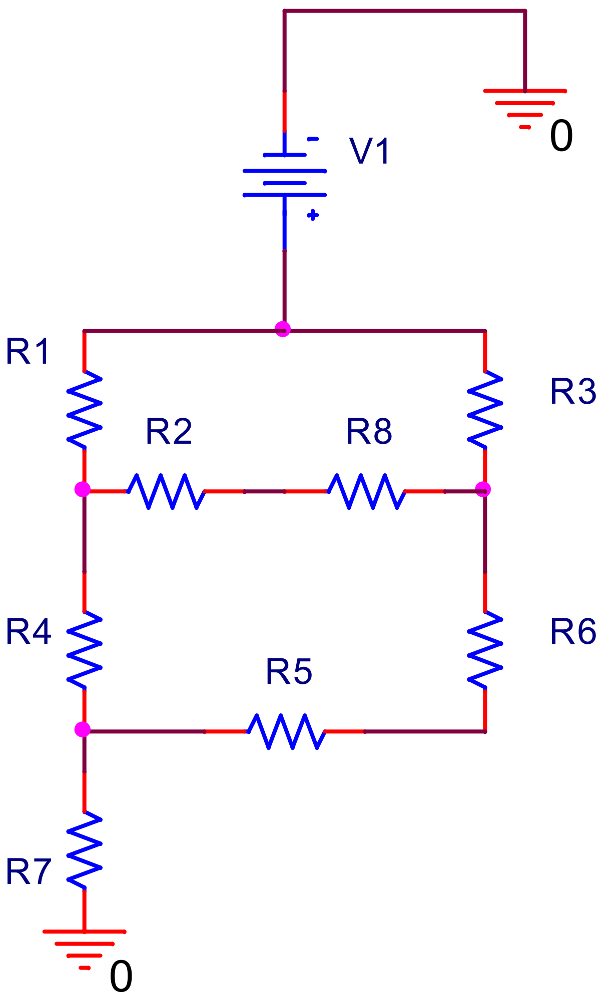
Equations for the transformation from Δ-load to Y-load

$$R_1 = \frac{R_a R_b}{R_a + R_b + R_c},$$
$$R_2 = \frac{R_b R_c}{R_a + R_b + R_c},$$
$$R_3 = \frac{R_a R_c}{R_a + R_b + R_c}.$$

Equations for the transformation from Y-load to Δ-load

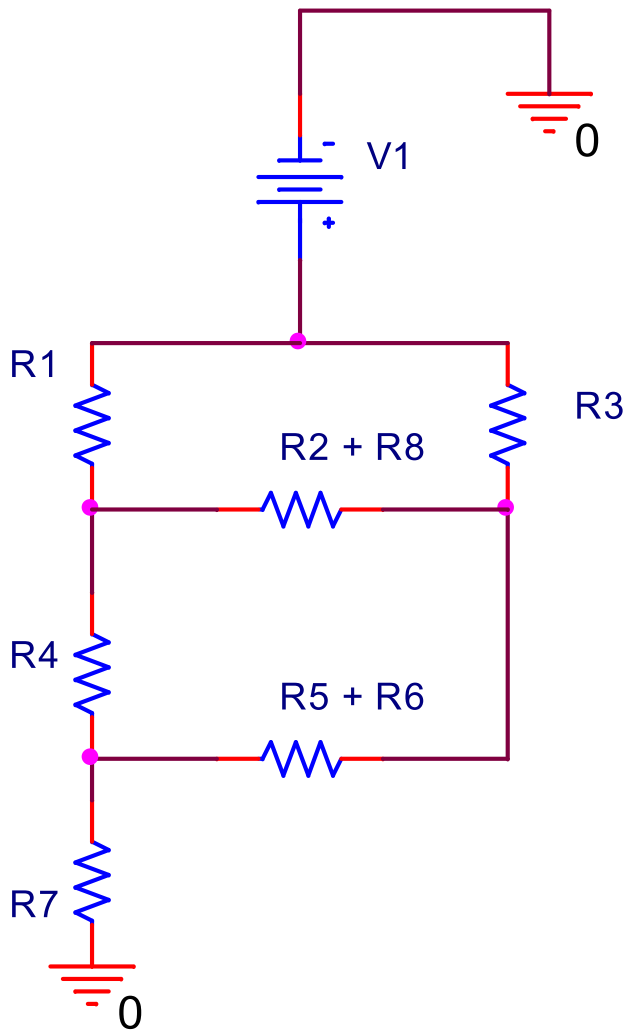
$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2},$$
$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3},$$
$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}.$$

Example

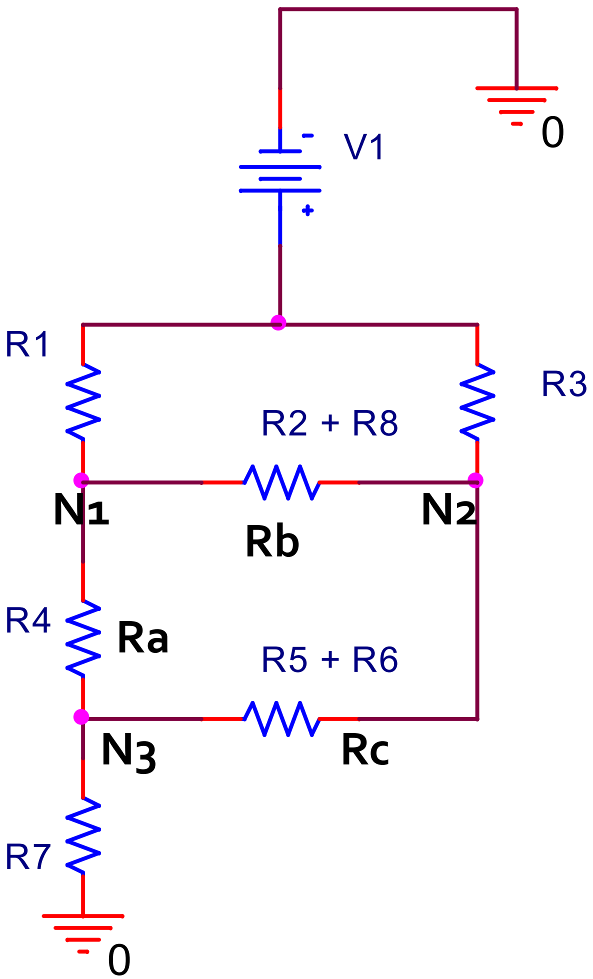


1. Realize that this network can't be simplified just through series and parallel simplifications

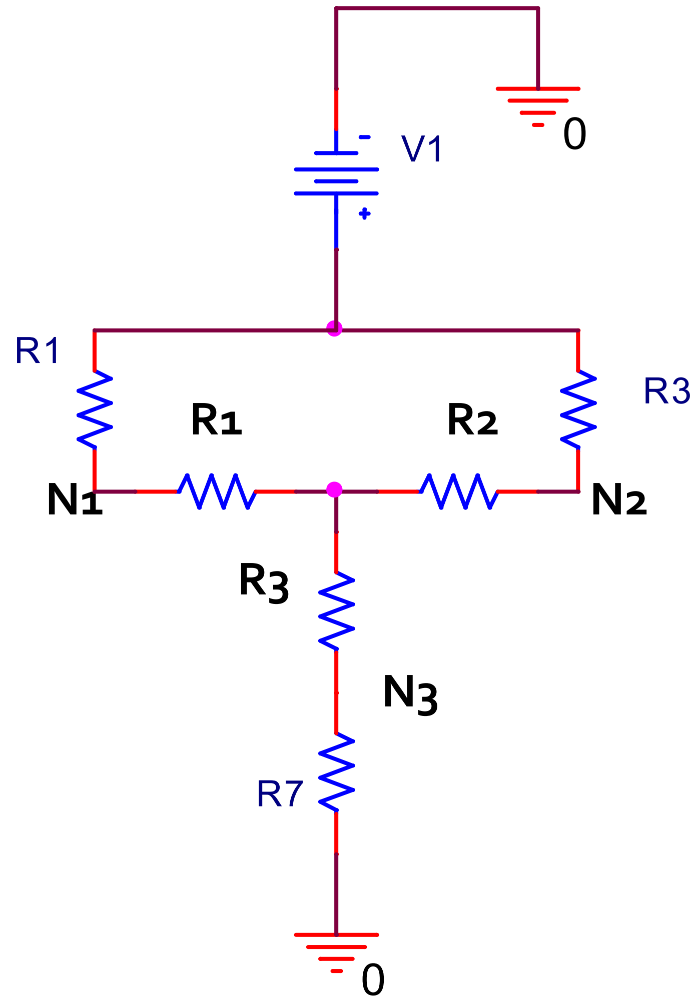
2. See R2 and R8 in series, and R5 and R6 in series



Example

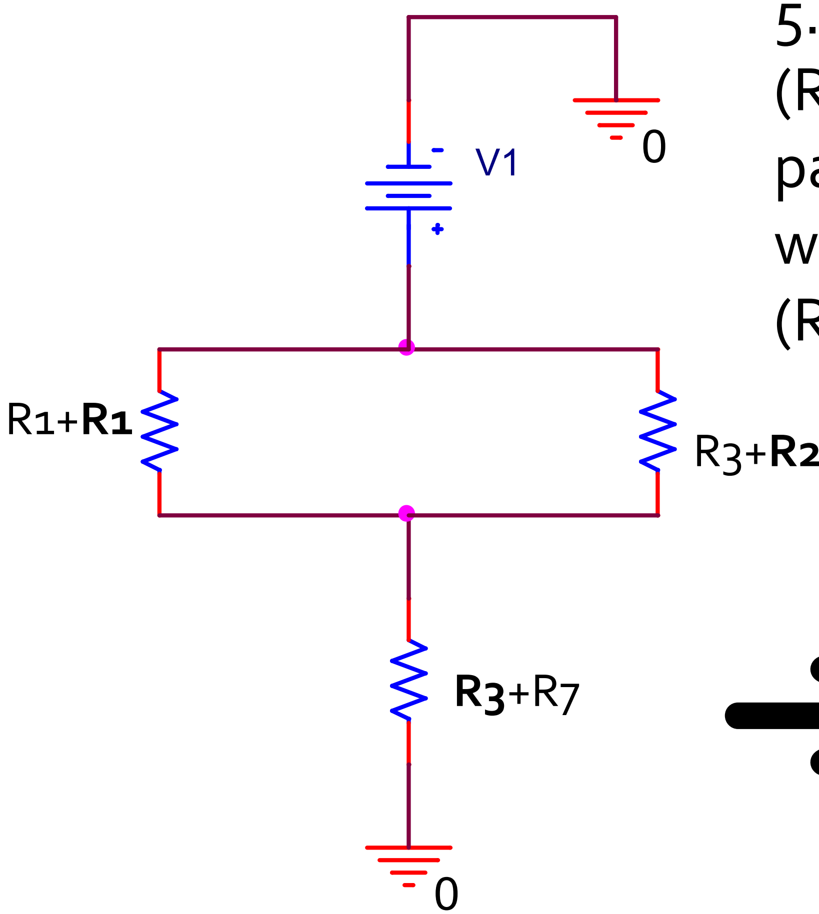


3. See that (R2+R8), (r5+R6), and R4 are in a delta configuration and that further analysis can be promoted by converting to wye configuration

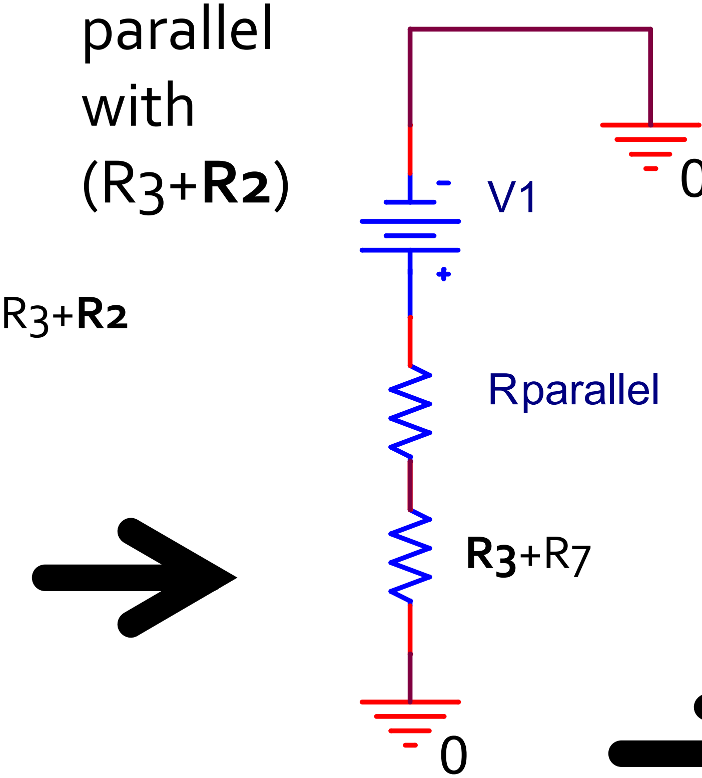


4. See R1 in series with **R1** and R3 in series with **R2** and **R3** in series with R7

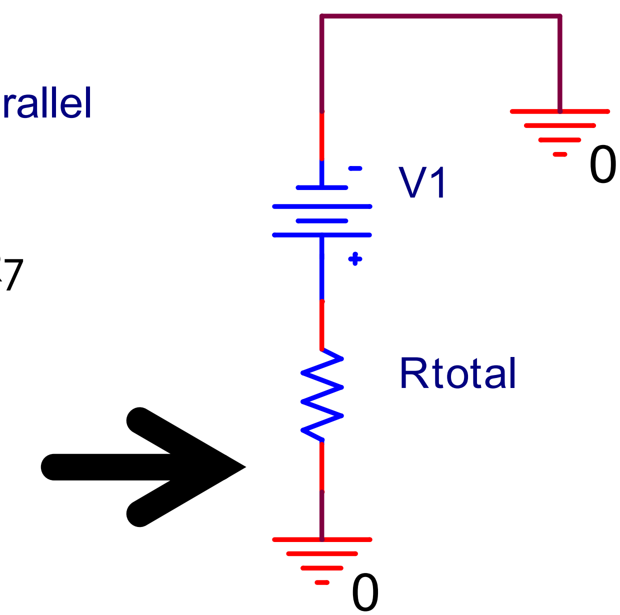
Example



5. See (R1+**R1**) in parallel with (R3+**R2**)



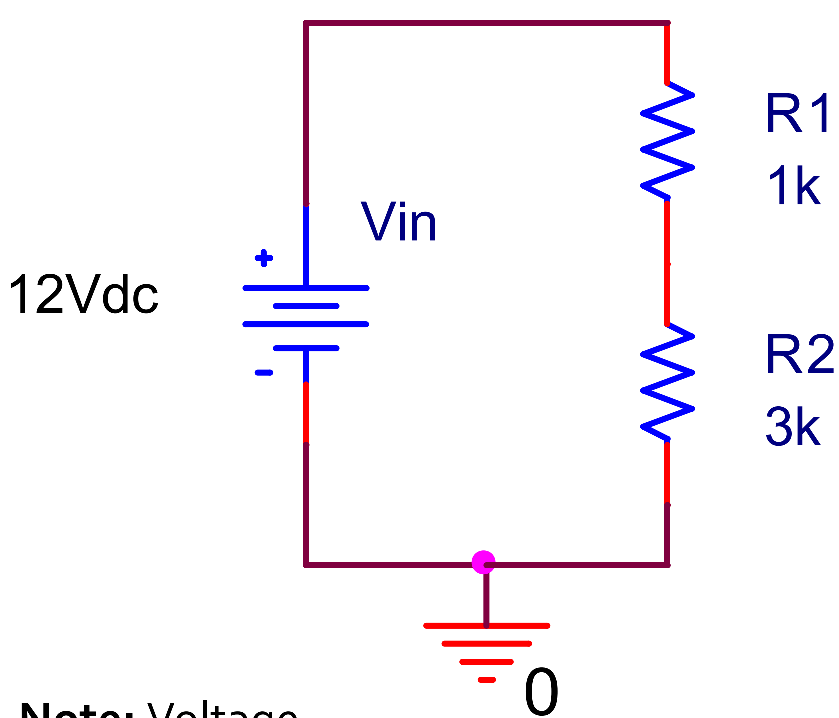
6. See $R_{parallel}$ in series with (**R3**+R7)



Voltage Divider

- **Key Idea:** The voltage that appears across a resistor – "dropped across it' – is proportional to the resistance
- Intuitive: If two resistors are in series, we expect that the one with more resistance will see the majority of the voltage
  - Since resistors in series have the same current through them, we can expect the component with greater resistance will "consume" most of the voltage

Voltage Divider



Voltage Divider Equation:

$V_{R2} = V_{in}*R_2/(R_1+R_2) =$
$12*3/(3+1) = 12V*3/4 = 9V$

$V_{R1} = V_{in}-V_{R2} =$
$V_{in}*R_1/(R_1+R_2) =$
$12V*1/(1+3) = 12V*1/4 = 3V$

Could also solve using KVL

Note: Voltage Divider Equation is simply a proportion!

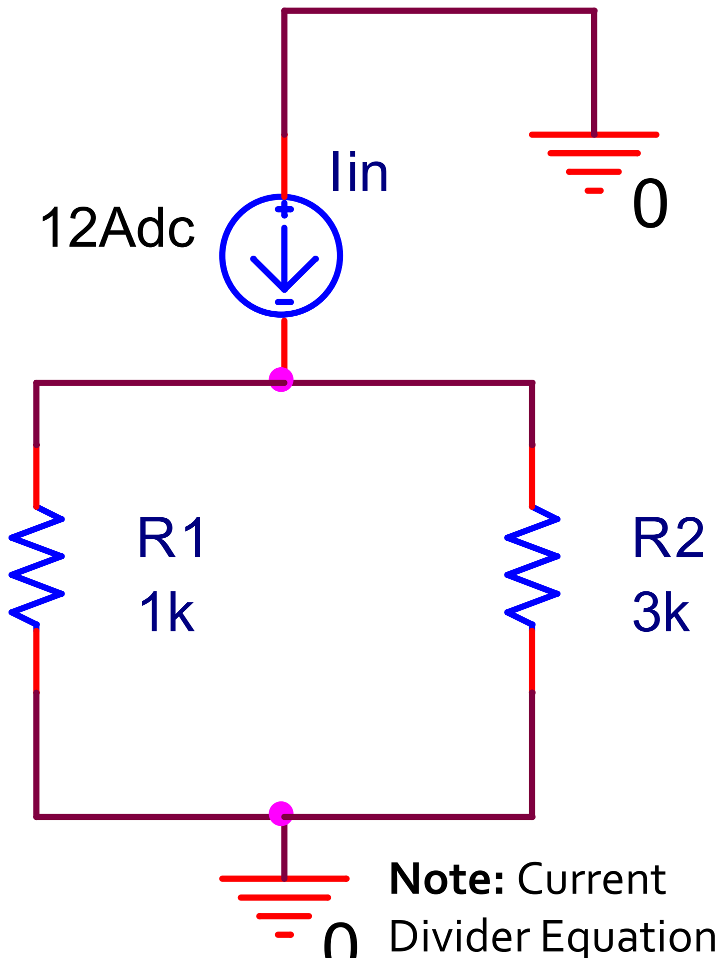
Voltage Divider = Pressure Divider

- In a microfluidic network, same rule holds:
  - Channels with high resistances (shallow, narrow, long) will have large pressure drop across them
  - Channels with low resistances (tall, wide, short) will have small pressure drop across them

Current Divider

- **Key Idea:** The current that flow through a resistor is inversely proportional to the resistance
- Intuitive: If two resistors are in parallel, we expect that most of the current will pass through the one with less resistance – "follow the path of least resistance"

Current Divider



Note: Current Divider Equation is simply a proportion!

Current Divider Equation:

i_{R1} =
$I_{in}*R_2/(R_1+R_2)=12A*3/(1+3)=9A$

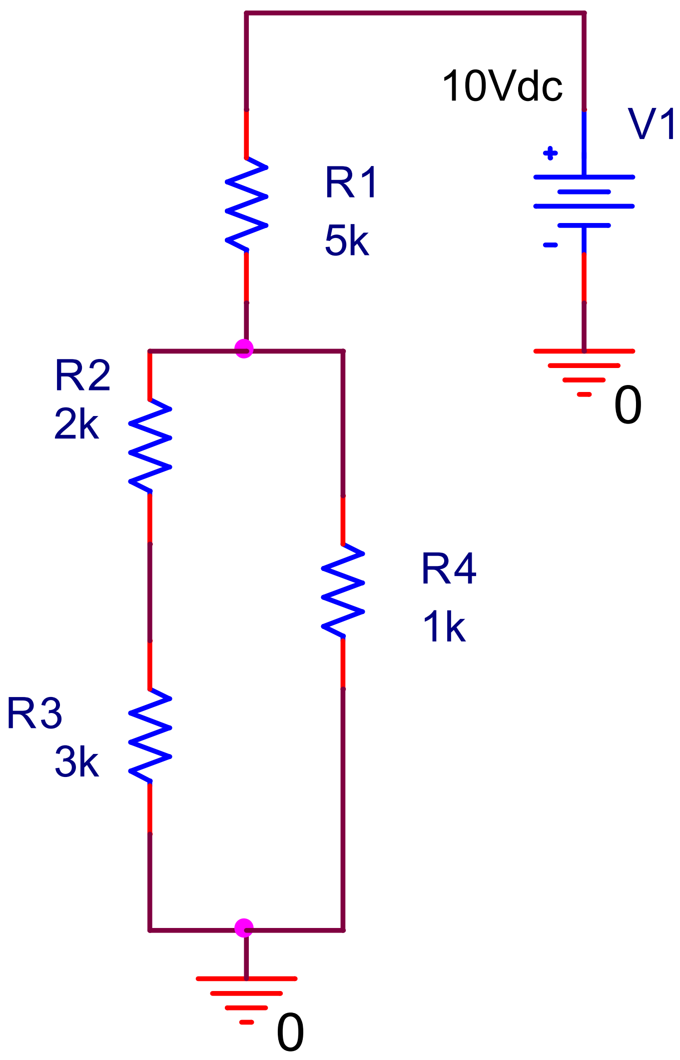
i_{R2} =
$I_{in}*R_1/(R_1+R_2)=12A*1/(1+3)=3A$

Could also solve using KCL:
$i_{R2} = I_{in}-i_{R1}$

Current Divider = Flow Divider

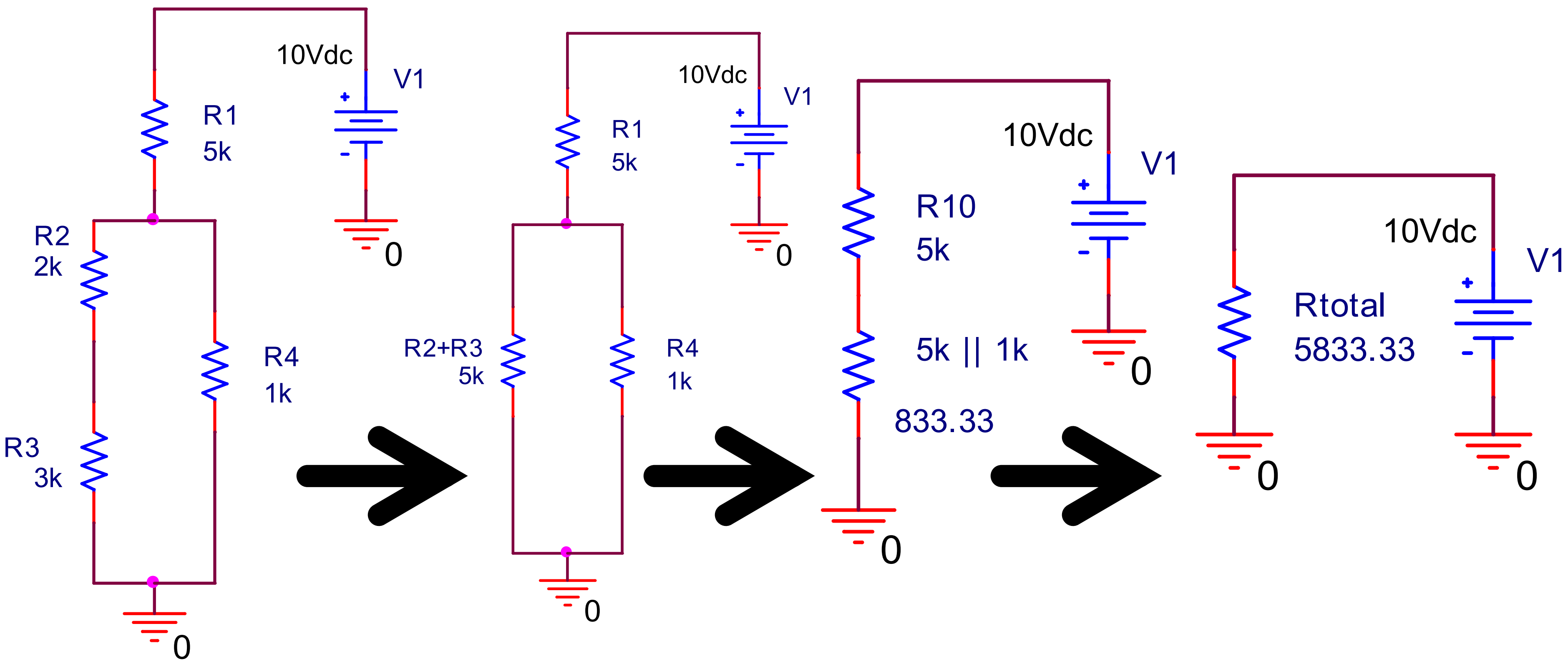
- In a microfluidic network, the same rule holds:
  - Channels with high resistances (shallow, narrow, long) will have small fluid flow through them
  - Channels with low resistances (tall, wide, short) will have large fluid flow through them

Composite Example



Find current through each resistor and voltage drop across each resistor

Solution: First, reduce to Rtotal



Assignment

- Several questions related to your lab and to lecture content
- **Remember:** linear velocity = volumetric flow rate / cross-sectional area of channel

Solution: Second, Find i_T

- Ohm's Law:
- V=iR
- I_T = V/R_T = 10V/5833.33Ω = **0.00171A = 1.71mA**

Solution: Third, find I and V for each resistor

Observation: Current through $R_1 = i_T$

Ohm's Law:
$V=iR$
$VR_1=i_T*R_1 = 1.71mA*5k\Omega = 8.57V$

Observation: V across (R_2+R_3) = V across R_4
KVL: $V_{R4} = 10V-8.57V = 1.43V$
Ohm's Law: $i_{R4} = V_{R4}/R_4 = 1.43V/1k\Omega = 1.43mA$

Observation: $i_{R2} = i_{R3}$
KCL: $i_{R2} = i_{R3} = i_{R1}-i_{R4} = 1.71mA-1.43mA = .28mA$
Ohm's Law: $V_{R2} = .28mA*2k\Omega = .56V$
Ohm's Law: $V_{R3} = .28mA*3k\Omega = .84V$